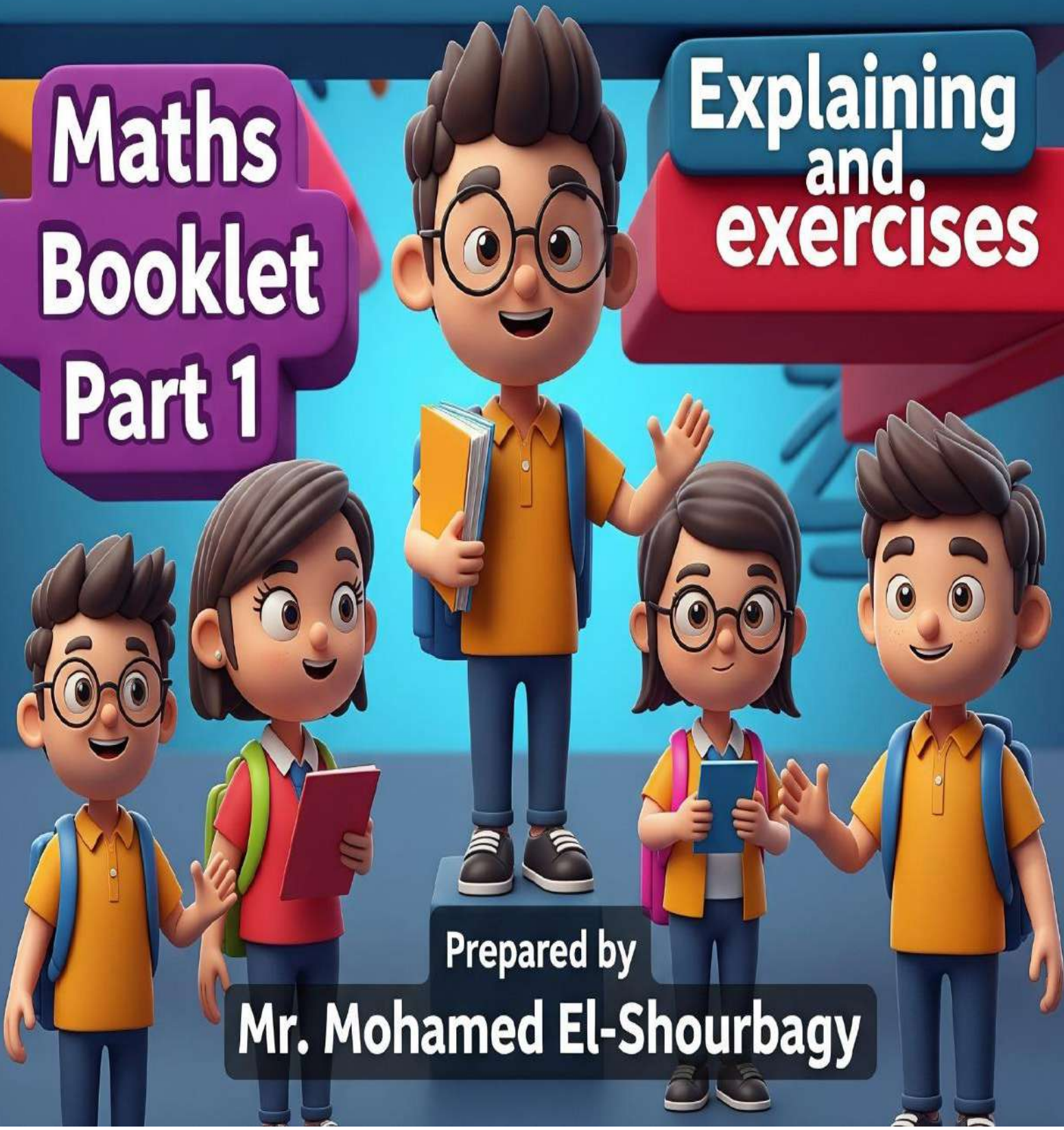


Simplest Maths Booklets

Secondary 1 - First Term 2026

**Maths
Booklet
Part 1**

**Explaining
and
exercises**



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SIMPLEST MATHS BOOKLETS

Contents

Unit 1 Pre-requirements on Unit One

Lesson 1
An introduction
to complex numbers

Lesson 3
Relation Between the
two roots of the
second degree
equation and the
coefficients of it terms

Lesson 5
Sign of a function

Lesson 2
Determining the
types of roots of
a quadratic equation

Lesson 4
Forming the quadratic
equation whose
two roots are known

Lesson 6
Quadratic
Inequality, in
one variable

Unit 3 SIMILARITY

1 Similarity of polygons.

2 Similarity of triangles

3 The relation between the
areas of two similar polygons

4 Applications of similarity
in the circle.

Unit 2 TRIGONOMETRY

1 Directed angle

2 Systems of measuring angle
(Degree measure - Radian measure)

3 Trigonometric functions

4 Related angles

5 Graphing trigonometric
functions

6 Finding the measure of
an angle given the value
of one of its trigonometric

Unit 4 TRIANGLE PROPORTIONALITY THEOREMS

Parallel Lines and
Proportional Parts

Thales' Theorem

Angle Bisector and
Proportional Parts

Converse of the
Angle Bisector Theorem

Simplest
Maths

EXPERIENCE
20
YEARS

دروس ماث
اونلاين و حضور

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للمدارس التجريبية واللغات الناشونال والانترناشونال

تقديمات اسبوعية وشهرية

متابعة مستمرة ومكافآت للمتفوقين

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اشترك شهري وغير القادرين مجاناً

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Unit 1

Pre-requirements on Unit One

Lesson 1

An introduction
to complex numbers

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Relation Between the
two roots of the
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SIMPLEST MATHS BOOKLETS

Pre-requirements on unit one

First Solving the quadratic equation in one variable algebraically

1 By factorization

Example 1

Find in $\mathbb{R}$ the solution set of each of the following equations :

1 $x^2 - 5x - 6 = 0$

2 $4x^2 = 25$

Solution

1 $\because x^2 - 5x - 6 = 0 \quad \therefore (x - 6)(x + 1) = 0$ "factorizing the trinomial"

$\therefore$ Either $x - 6 = 0$ or $x + 1 = 0$

$\therefore x = 6$ or $x = -1$

$\therefore$ The solution set = $\{6, -1\}$

2 $\because 4x^2 = 25 \quad \therefore 4x^2 - 25 = 0$

$\therefore (2x - 5)(2x + 5) = 0$ "factorizing the difference between two squares"

$\therefore$ Either $2x - 5 = 0$ or $2x + 5 = 0$

$\therefore x = \frac{5}{2}$ or $x = -\frac{5}{2}$

$\therefore$ The solution set = $\{\frac{5}{2}, -\frac{5}{2}\}$

Remember that

The quadratic equation in one variable has at most two solutions in $\mathbb{R}$

Another solution

$\because 4x^2 = 25 \quad \therefore x^2 = \frac{25}{4} \quad \therefore x = \pm \sqrt{\frac{25}{4}}$
 $\therefore x = \pm \frac{5}{2} \quad \therefore$ The solution set = $\{\frac{5}{2}, -\frac{5}{2}\}$

2 By the general formula

To find the roots of the quadratic equation : $ax^2 + bx + c = 0$ where $a \neq 0$

use the formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Example 2

Find the solution set of each of the following equations in $\mathbb{R}$:

1 $x^2 - 2x - 6 = 0$

2 $x + \frac{5}{x} = 4, x \neq 0$

Solution

1 The expression : $x^2 - 2x - 6$ is difficult to be factorized , so we use the general formula.

$\therefore a = 1, b = -2, c = -6$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \times 1 \times (-6)}}{2 \times 1}$

$= \frac{2 \pm \sqrt{4 + 24}}{2} = \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$

$\therefore$ The solution set = $\{1 + \sqrt{7}, 1 - \sqrt{7}\}$

2 $\because x + \frac{5}{x} = 4$ "By multiplying both sides of the equation by x "

$\therefore x^2 + 5 = 4x$

$\therefore x^2 - 4x + 5 = 0$ "Notice putting the equation in the form : $ax^2 + bx + c = 0$ "

$\therefore a = 1, b = -4, c = 5$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{16 - 4 \times 1 \times 5}}{2} = \frac{4 \pm \sqrt{-4}}{2}$

$\therefore \because \sqrt{-4} \notin \mathbb{R} \quad \therefore$ There is no real roots of the equation : $x^2 - 4x + 5 = 0$

$\therefore$ The solution set = $\emptyset$

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Second Solving the quadratic equation in one variable graphically

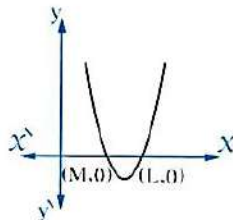
To solve the quadratic equation in one variable graphically , we do the following :

- 1 Put the equation on the form : $aX^2 + bX + c = 0$
- 2 Let $f(X) = aX^2 + bX + c$
- 3 Graph the function f
- 4 Determine the points of intersection of the curve with the X -axis , then the X -coordinates of these intersection points are the solutions of the equation $f(X) = 0$ i.e. $aX^2 + bX + c = 0$

According to that , we have three cases

1

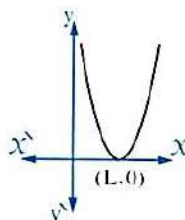
The curve intersects X -axis at two points



There are two solutions
in $\mathbb{R}$
The S.S. = $\{L, M\}$

2

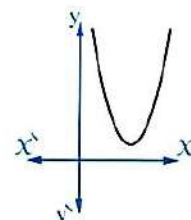
The curve touches X -axis at one point



There is a unique solution
in $\mathbb{R}$
The S.S. = $\{L\}$

3

The curve does not intersect X -axis



There is no solution
in $\mathbb{R}$
The S.S. = $\emptyset$

Example 3

Find graphically in $\mathbb{R}$ the S.S. of the equation :

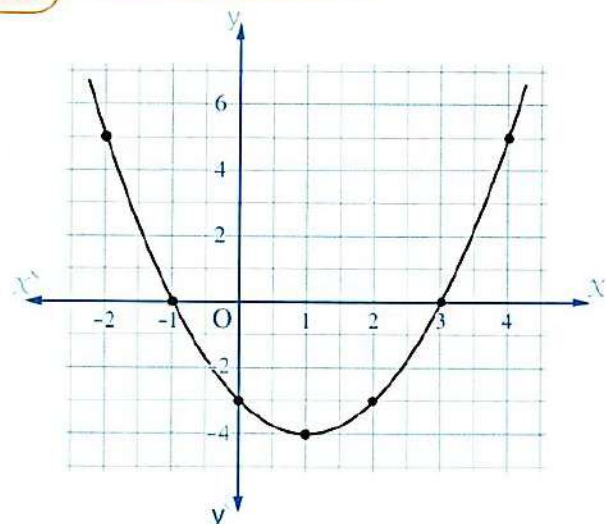
$X^2 - 2X - 3 = 0$ using the interval $[-2, 4]$

Solution

Let $f(X) = X^2 - 2X - 3$

X	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

From the graph , the S.S. = $\{3, -1\}$



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Remark

In case of the interval is not given , then we can graph the function by finding the vertex of the curve which is $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$, and then we find some points to the right of it , and the same number of points to the left of it.

Example 4

Solve graphically in $\mathbb{R}$ the equation :

$4x(x-1)-5=0$, then verify the result algebraically “given that $\sqrt{6} \simeq 2.4$ ”

Solution

$$\therefore 4x(x-1)-5=0 \quad \therefore 4x^2-4x-5=0$$

First Graphically :

$$\text{Let } f(x) = 4x^2 - 4x - 5$$

• Find the vertex point of the curve :

$$\therefore \text{The } x\text{-coordinate of the vertex point} = \frac{-b}{2a} = \frac{4}{8} = \frac{1}{2}$$

$$, f\left(\frac{1}{2}\right) = 4\left(\frac{1}{2}\right)^2 - 4\left(\frac{1}{2}\right) - 5 = -6$$

$$\therefore \text{The vertex point of the curve is } \left(\frac{1}{2}, -6\right)$$

• Form the following table :

x	-1	0	$\left(\frac{1}{2}\right)$	1	2
y	3	-5	(-6)	-5	3

• From the graph we notice that :

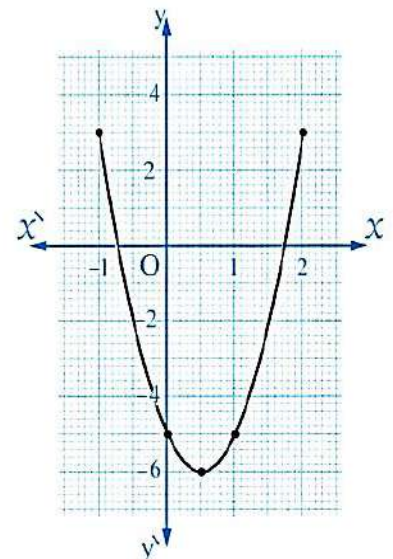
The roots are -0.7 and 1.7 approximately.

Second Algebraically :

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \text{ where } a = 4, b = -4, c = -5$$

$$\therefore x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \times 4 \times (-5)}}{2 \times 4} = \frac{4 \pm \sqrt{96}}{8} = \frac{4 \pm 4\sqrt{6}}{8} = \frac{1 \pm \sqrt{6}}{2} \simeq \frac{1 \pm 2.4}{2}$$

$\therefore$ The two roots of the equation are 1.7 and -0.7 approximately.



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First Multiple choice questions

Choose the right answer from those given :

(1) The solution set of the equation : $x^2 - 1 = 0$ in $\mathbb{R}$ is

- (a) $\emptyset$ (b) 1 (c) ± 1 (d) $\{1, -1\}$

(2) The solution set of the equation : $x^2 - 6x + 9 = 0$ in $\mathbb{R}$ is

- (a) $\{-3\}$ (b) $\{3\}$ (c) $\emptyset$ (d) $\{9\}$

(3) The solution set of the equation : $x^2 - x = 0$ in $\mathbb{R}$ is

- (a) $\{1, -1\}$ (b) $\{0\}$ (c) $\{0, 1\}$ (d) $\emptyset$

(4) The solution set of the equation : $x^2 + 3x = 0$ in $\mathbb{R}^*$ is

- (a) $\{0, -3\}$ (b) $\emptyset$ (c) $(0, 3)$ (d) $\{-3\}$

(5) Number of roots of the equation : $x^2 + 9 = 0$ in $\mathbb{R}$ is

- (a) 2 (b) 1 (c) 3 (d) zero

(6) The necessary condition which makes the equation $ax^2 + bx + c = 0$ quadratic is

- (a) $a > 0$ (b) $a < 0$ (c) $a \neq 0$ (d) $a \neq 0, b \neq 0$

(7) The two quadratic equations : $x^2 - 3x + 2 = 0$ and $2x^2 - 5x + 2 = 0$ have common solution is

- (a) $x = 2$ (b) $x = 1$ (c) $x = -2$ (d) $x = \frac{1}{2}$

SIMPLEST MATHS BOOKLETS

Second / Essay questions

- 1** Find in $\mathbb{R}$ the solution set of each of the following equations by using the general formula approximating the result to the nearest tenth :

$$(1) x^2 - 6x + 1 = 0$$

$$(2) x^2 + 3x + 5 = 0$$

$$(3) \square 2x^2 + 3x - 4 = 0$$

$$(4) \square 3x^2 - 65 = 0$$

- 2** Find in $\mathbb{R}$ the solution set of each of the following equations algebraically , then check the answer graphically :

$$(1) x^2 - 2x - 4 = 0$$

(Hint : draw graphically in the interval $[-2, 4]$)

$$(2) 3x - x^2 + 2 = 0$$

(Hint : draw graphically in the interval $[-1, 4]$)

- 3** $\square$ If the sum of the whole consecutive numbers $(1 + 2 + 3 + \dots + n)$ is given by the relation $S = \frac{n}{2} (1 + n)$, how many whole consecutive numbers starting from the number 1 and their sum equals :

$$(1) 78$$

$$(2) 171$$

$$(3) 253$$

$$(4) 465$$

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Lesson

1

An introduction in complex numbers

Introduction

- There are many problems that can not be solved by the use of real numbers alone. For example, we are unable to solve the equation $X^2 = -1$. There is no real number "a" such that $a^2 = -1$. Thus we must extend the set of real numbers $\mathbb{R}$ to a new set of numbers to enable us to find the solution of the equation $X^2 = -1$.

This new set is called **THE SET OF COMPLEX NUMBERS**, and before studying the set of complex numbers in details, we will firstly recognize the imaginary number "i".

The imaginary number "i"

The imaginary number "i" is defined as the number whose square is -1

i.e. $i^2 = -1$

Thus we can solve the equation : $X^2 = -1$ as follows :

$$\therefore X^2 = -1$$

$$\therefore X^2 = i^2$$

$$\therefore X = \pm \sqrt{i^2}$$

$$\therefore X = \pm i$$

$$\therefore \text{The solution set} = \{i, -i\}$$

Notice that

- $i \times i = i^2 = -1$
- $-i \times -i = i^2 = -1$

Remarks

- ▶ The number "i" does not belong to the set of real numbers.

i.e. $i \notin \mathbb{R}$, so it will not be represented by a point on the real number line.

- ▶ The numbers $3i, -2i, \sqrt{5}i, \dots$ are imaginary numbers.

- ▶ If a is a real positive number, then $\sqrt{-a} = \sqrt{a}i$

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Remark

We can express "1" using the imaginary number i to integer powers from the multiples of 4 , and this helps in simplyfying some of imaginary numbers , for example : $i^{-19} = \frac{1}{i^{19}} = \frac{i^{20}}{i^{19}} = i$

The complex number

The complex number is the number that can be written in the form $a + b i$, where a and b are two real numbers and $i^2 = -1$

- a is called the real part.
- b is called the imaginary part.

Examples for complex numbers : $2 - i$, $7 + 13 i$, $5 i - 4$, $\sqrt{2} + \sqrt{3} i$

Remarks

For any complex number $(Z = a + b i)$, then :

1 If $(b = 0)$, then $Z = a$ and we say that Z is a real number.

Such as $Z = 5$ is a real number and it is a complex number whose imaginary number = 0

2 If $(a = 0)$, then $Z = b i$ and we say that Z is an imaginary number. (where $b \neq 0$)

Such as $Z = 2 i$ is an imaginary number and it is a complex number.

From the previous , every real number is a complex number whose imaginary number = zero and so the set of real numbers is a subset of set of complex numbers that can be defined as the following.

The set of complex numbers

The set of complex numbers $\mathbb{C}$ is defined as $\mathbb{C} = \{a + b i : a \in \mathbb{R} , b \in \mathbb{R} , i^2 = -1\}$

Example 1

Find the solution set of each of the following equations in the set of complex numbers :

1 $2x^2 + 18 = 0$

2 $x^2 + x + 1 = 0$

Solution

1 $\therefore 2x^2 + 18 = 0 \quad \therefore 2x^2 = -18 \quad \therefore x^2 = -9$

$\therefore x = \pm\sqrt{-9} \quad \therefore x = \pm\sqrt{9i^2} \quad \therefore x = \pm 3i$

$\therefore$ The solution set = $\{3i , -3i\}$

2 $\therefore a = 1 , b = 1 , c = 1$

$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-1 \pm \sqrt{1^2 - 4 \times 1 \times 1}}{2 \times 1} = \frac{-1 \pm \sqrt{-3}}{2} = \frac{-1 \pm \sqrt{3}i}{2} = \frac{-1}{2} \pm \frac{\sqrt{3}}{2}i$

$\therefore$ The solution set = $\left\{\frac{-1}{2} + \frac{\sqrt{3}}{2}i , \frac{-1}{2} - \frac{\sqrt{3}}{2}i\right\}$

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Equality of two complex numbers

Two complex numbers are equal if and only if the two real parts are equal and the two imaginary parts are equal.

i.e. If $(a + b i)$ and $(c + d i)$ are two complex numbers and if $a = c$, $b = d$
 , then $a + b i = c + d i$

and vice versa If $a + b i = c + d i$, then $a = c$, $b = d$

Notice that Order in complex numbers whose imaginary part not equal to zero has no meaning , we do not know which is greater $(5 + 3 i)$ or $(-4 + 7 i)$?

Example 2

Find the values of x and y which satisfy each of the following where $x \in \mathbb{R}$, $y \in \mathbb{R}$, $i^2 = -1$:

1 $(2x - 3) + 5i = 7 + (3 - 2y)i$

2 $x + yi = \sqrt{-4} + i^{22}$

3 $x - 3y + (2x + y)i = 6 + 5i$

Solution

1 $\therefore 2x - 3 = 7$

$\therefore 2x = 10$

$\therefore x = 5$

, $\therefore 3 - 2y = 5$

$\therefore -2y = 2$

$\therefore y = -1$

2 $x + yi = 2i + i^{4(5)+2}$

$\therefore x + yi = 2i + i^2 = 2i + (-1)$

$\therefore x + yi = -1 + 2i$

$\therefore x = -1$, $y = 2$

3 $\therefore x - 3y = 6$

(1)

, $2x + y = 5$

(2)

Multiply the equation (2) by 3

$\therefore 6x + 3y = 15$

(3)

By adding (1) and (3) :

$\therefore 7x = 21$

$\therefore x = 3$

By substituting in (2) :

$\therefore y = -1$

SIMPLEST MATHS BOOKLETS

Adding and subtracting complex numbers

- When adding or subtracting two complex numbers, we add or subtract real parts together and add or subtract imaginary parts together.

Example 3

Find the result of each of the following in the simplest form :

1 $(3 + 7i) + (5 - 9i)$

2 $(2 - \sqrt{-16}) - (5 - i)$

Solution

1 $\because i^2 = -1$ $\therefore$ The expression $= (3 + 7i) + (5 - 9i) = (3 + 5) + (7i - 9i) = 8 - 2i$

2 $\because \sqrt{-16} = 4i$

$\therefore$ The expression $= (2 - 4i) - (5 - i) = (2 - 4i) + (-5 + i) = (2 - 5) + (-4i + i) = -3 - 3i$

Multiplying complex numbers

Two complex numbers can be multiplied just as the algebraic expressions, considering $i^2 = -1$

Example 4

Find the result of each of the following in the simplest form :

1 $(4 + 3i)(2 - 5i)$

2 $(5 - 2i)(5 + 2i)$

3 $(3 + 2i)^2$

4 $(1 - i)^4$

Solution

$$\begin{aligned} 1 \quad (4 + 3i)(2 - 5i) &= 4(2 - 5i) + 3i(2 - 5i) \\ &= 8 - 20i + 6i - 15i^2 \\ &= 8 - 20i + 6i + 15 \quad (\text{where } i^2 = -1) \\ &= (8 + 15) + (-20i + 6i) = 23 - 14i \end{aligned}$$

Notice that You can solve directly by using multiplication by inspection as follows :

$$\begin{array}{r} -20i \\ (4 + 3i)(2 - 5i) = 8 - 14i - 15i^2 \quad (\text{where } i^2 = -1) \\ +6i \quad \quad \quad = 8 - 14i + 15 = 23 - 14i \end{array}$$

$$\begin{aligned} 2 \quad (5 - 2i)(5 + 2i) &= 25 - 4i^2 \\ &= 25 + 4 \quad (\text{where } i^2 = -1) \\ &= 29 \end{aligned}$$

$$\begin{aligned} 3 \quad (3 + 2i)^2 &= 9 + 12i + 4i^2 \\ &= 9 + 12i - 4 \quad (\text{where } i^2 = -1) \\ &= 5 + 12i \end{aligned}$$

$$\begin{aligned} 4 \quad (1 - i)^4 &= ((1 - i)^2)^2 = (1 - 2i + i^2)^2 = (1 - 2i - 1)^2 \\ &= (-2i)^2 = 4i^2 = -4 \end{aligned}$$

Remember that

$$(a + b)(a - b) = a^2 - b^2$$

Remember that

$$(a \pm b)^2 = a^2 \pm 2ab + b^2$$

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Remark

$$(1 \pm i)^{2n} = (\pm 2i)^n \text{ where } n \in \mathbb{Z}$$

- **Proof :** $(1 \pm i)^{2n} = [(1 \pm i)^2]^n = [1 \pm 2i - 1]^n = (\pm 2i)^n$
- This remark is used to simplify some complex numbers as the following :

$$\text{① } (1 + i)^{200} = (2i)^{100} = 2^{100} i^{100} = 2^{100}$$

$$\text{② } (3 - 3i)^4 = 3^4 (1 - i)^4 = 3^4 (-2i)^2 = 3^4 \times 2^2 i^2 = -324$$

The two conjugate numbers

The two numbers $a + bi$ and $a - bi$ are called conjugate numbers.

Note : Take care that the complex number and its conjugate differ only in the sign of their imaginary parts.

For example : The two numbers $3 + 4i$, $3 - 4i$ are conjugate numbers.

Remarks

- ▶ The conjugate of the number $2i - 5$ is the number $-2i - 5$ not $2i + 5$
- ▶ The conjugate of the number $2i$ is $-2i$
- ▶ The conjugate of the number 3 is 3
- ▶ The sum of the two conjugate numbers is always a real number , and the product of the two conjugate numbers is always a real number.

For example The complex number $3 + 4i$ its conjugate is $3 - 4i$, then :

$$* \text{ Their sum} = (3 + 4i) + (3 - 4i) = (3 + 3) + (4i - 4i) = 6 \in \mathbb{R}$$

$$* \text{ Their product} = (3 + 4i)(3 - 4i) = 9 - 16i^2 = 9 + 16 = 25 \in \mathbb{R}$$

Example 5

Simplify to the simplest form :

$$1 \frac{4 - 3i}{i}$$

$$2 \frac{10}{3 + i}$$

$$3 \frac{3 + 2i}{2 - 5i}$$

$$4 \frac{(2 + i)(1 - i)}{(1 + i)(3 - 2i)}$$

Solution

Notice : To simplify the fraction whose denominator is a complex number , we multiply its two terms by the conjugate of denominator.

$$1 \frac{4 - 3i}{i} \times \frac{-i}{-i} = \frac{-4i + 3i^2}{-i^2} = \frac{-4i - 3}{-(-1)} = -3 - 4i$$

$$2 \because \text{The conjugate of the denominator is } (3 - i)$$

$$\therefore \frac{10}{3 + i} = \frac{10(3 - i)}{(3 + i)(3 - i)} = \frac{10(3 - i)}{9 - i^2} = \frac{10(3 - i)}{9 + 1} = \frac{10(3 - i)}{10} = 3 - i$$

$$3 \frac{3 + 2i}{2 - 5i} = \frac{(3 + 2i)(2 + 5i)}{(2 - 5i)(2 + 5i)} = \frac{6 + 15i + 4i + 10i^2}{4 - 25i^2}$$

$$= \frac{6 + 19i - 10}{4 + 25} = \frac{-4 + 19i}{29} = \frac{-4}{29} + \frac{19}{29}i$$

$$4 \frac{(2 + i)(1 - i)}{(1 + i)(3 - 2i)} = \frac{2 - 2i + i - i^2}{3 - 2i + 3i - 2i^2} = \frac{2 - i + 1}{3 + i + 2} = \frac{3 - i}{5 + i}$$

$$, \frac{3 - i}{5 + i} = \frac{(3 - i)(5 - i)}{(5 + i)(5 - i)} = \frac{15 - 8i - 1}{25 - i^2} = \frac{14 - 8i}{26} = \frac{2(7 - 4i)}{26} = \frac{7}{13} - \frac{4}{13}i$$

SIMPLEST MATHS BOOKLETS

Example 6

If $x = \frac{7-i}{2-i}$ and $y = \frac{13-i}{4+i}$

Prove that : x and y are conjugate numbers , then prove that : $x^2 + y^2 = 16$

Solution

$$\therefore x = \frac{7-i}{2-i} = \frac{(7-i)(2+i)}{(2-i)(2+i)} = \frac{14+7i-2i-i^2}{4-i^2} = \frac{14+5i+1}{4+1} = \frac{15+5i}{5} = 3+i$$

$$, y = \frac{13-i}{4+i} = \frac{(13-i)(4-i)}{(4+i)(4-i)} = \frac{52-13i-4i+i^2}{16-i^2} = \frac{52-17i-1}{16+1} = \frac{51-17i}{17} = 3-i$$

$\therefore x$ and y are conjugate numbers " Notice that the signs of the imaginary parts are different."

$$, x^2 = (3+i)^2 = 9+6i+i^2 = 8+6i$$

$$, y^2 = (3-i)^2 = 9-6i+i^2 = 8-6i$$

$$\therefore x^2 + y^2 = (8+6i) + (8-6i) = (8+8) + (6i-6i) = 16$$

SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

1	Which of the following is an imaginary number ? (a) π (b) $\sqrt{5}$ (c) $\sqrt{-5}$ (d) i^2
2	$i^{24} = \dots\dots\dots$ (a) -1 (b) i^9 (c) $-i$ (d) 1
3	The simplest form of the imaginary number i^{45} is $\dots\dots\dots$ (a) i (b) -1 (c) $-i$ (d) 1
4	$i^{-30} = \dots\dots\dots$ (a) 1 (b) -1 (c) $-i$ (d) i
5	$\frac{1}{i^{199}} = \dots\dots\dots$ (a) 1 (b) $-i$ (c) i (d) -1
6	$i^{26} + i^{28} = \dots\dots\dots$ (a) i^{54} (b) $-i$ (c) zero (d) 2
7	$\frac{1}{i^{15}} + i^{21} = \dots\dots\dots$ (a) zero (b) $2i$ (c) $-2i$ (d) $-i$
8	$5i^7 + 4i^{-1} = \dots\dots\dots$ (a) $9i$ (b) $-9i$ (c) i (d) $-i$
9	$1 + i + i^2 + i^3 + i^4 = \dots\dots\dots$ (a) $4i + 1$ (b) -1 (c) 1 (d) 5
10	If $n \in \mathbb{Z}$, then $i^{8n-3} = \dots\dots\dots$ (a) i (b) $-i$ (c) -1 (d) 1
11	If $n \in \mathbb{Z}$, then $i^{4n+42} = \dots\dots\dots$ (a) 1 (b) -1 (c) $-i$ (d) i
12	The additive inverse of the complex number $(4 - 7i)$ is $\dots\dots\dots$ (a) $4 + 7i$ (b) $-4 + 7i$ (c) $-4 - 7i$ (d) $4 - 7i$

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13	The conjugate of the number $(3i - 4)$ is
	(a) $3i + 4$ (b) $-3i - 4$ (c) $-3i + 4$ (d) $3i - 4$
14	The conjugate of the number $(i - i^2)$ is
	(a) $1 - i$ (b) $1 + i$ (c) $-i - 1$ (d) $i - 1$
15	The conjugate of the number (-8) is
	(a) $8i$ (b) $-8i$ (c) -8 (d) 8
16	The conjugate of the number $(2 + i)^2$ is
	(a) $2 + i$ (b) $(2 + i)^{-1}$ (c) $3 + 4i$ (d) $3 - 4i$
17	$\sqrt{2} \times \sqrt{-8} = \dots\dots\dots$
	(a) i (b) $-2i$ (c) $4i$ (d) $-4i$
18	$\sqrt{-18} \times \sqrt{-12} = \dots\dots\dots$
	(a) $6\sqrt{6}i$ (b) $6\sqrt{6}$ (c) $-6\sqrt{6}$ (d) $-6\sqrt{6}i$
19	$\sqrt{-9} \times \sqrt{\frac{-1}{9}} = \dots\dots\dots$
	(a) i (b) $-i$ (c) -1 (d) 1
20	$(-4i)(-6i) = \dots\dots\dots$
	(a) $-10i$ (b) $24i$ (c) $-24i$ (d) -24
21	$(-2i)^3(-3i)^2 = \dots\dots\dots$
	(a) $-72i$ (b) $72i$ (c) 72 (d) -72
22	$(3 + 2i) + (2 - 5i) = \dots\dots\dots$
	(a) $5 + 2i$ (b) $5 - 3i$ (c) $3 - 5i$ (d) $5 + 3i$
23	If x, y are real numbers and $(2 + 5i) - (4 - 2i) = x + yi$, then $x + y = \dots\dots\dots$
	(a) 9 (b) -1 (c) 1 (d) 5

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Ex.2) Answer the following:

1 Find the result of each of the following in the simplest form :

$$(1) \text{ (book icon) } (2 + \sqrt{-9})(3 - 4i)$$

$$(2) (2 - 5i)^2$$

$$(3) (3 - 2i)^2 + (3 + 2i)$$

$$(4) (1 + i)^4$$

$$(5) (1 + \sqrt{-1})^4 - (1 - \sqrt{-1})^4$$

$$(6) \text{ (book icon) } (1 - i)^{10}$$

2 Put each of the following in the form $(a + bi)$ where a and b are real numbers :

$$(1) \frac{4 - 5i}{7i}$$

$$(2) \text{ (book icon) } \frac{26}{3 - 2i}$$

$$(3) \text{ (book icon) } \frac{2 - 3i}{3 + i}$$

$$(4) \text{ (book icon) } \frac{3 + 4i}{5 - 2i}$$

$$(5) \frac{(3 + 2i)(2 - i)}{3 + i}$$

$$(6) \text{ (book icon) } \frac{(3 + i)(3 - i)}{3 - 4i}$$

3  Solve each of the following equations in the set of complex numbers :

$$(1) 3x^2 + 12 = 0$$

$$(2) 4x^2 + 100 = 75$$

$$(3) x^2 - 4x + 5 = 0$$

$$(4) 2x^2 + 6x + 5 = 0$$

4 Find the values of x and y that satisfy each of the following equations where x and y are real numbers :

$$(1) \text{ (book icon) } (2x - 3) + (3y + 1)i = 7 + 10i$$

$$(2) \text{ (book icon) } (2x - y) + (x - 2y)i = 5 + i$$

$$(3) 3x + xi - 2y + yi = 5$$

$$(4) \text{ (book icon) } \frac{(2 + i)(2 - i)}{3 + 4i} = x + yi$$

5 If $x = \frac{13}{5 - i}$, $y = \frac{3 + 2i}{1 + i}$
 , prove that : x and y are two conjugate numbers.

6 If a, b are two real numbers and $a + bi = \frac{2+i}{2-i}$, prove that : $a^2 + b^2 = 1$

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Higher skills

1 Choose the correct answer from those given :

- (1) If L, M are the roots of a quadratic equations : $X^2 + 1 = 0$, then $L^{2018} + M^{2018} = \dots\dots\dots$
(a) $-2i$ (b) $2i$ (c) -2 (d) 2018
- (2) $(1 + i)^{2020} = \dots\dots\dots$
(a) $(1 - i)^{2020}$ (b) 2^{1010} (c) $2^{1010} i$ (d) i^{2020}
- (3) If $\left(\frac{1-i}{1+i}\right)^{100} = X + yi$, then $(X, y) = \dots\dots\dots$
(a) $(0, 1)$ (b) $(-1, 0)$ (c) $(0, -1)$ (d) $(1, 0)$
- (4) The conjugate of the number $(2 + i)^{-1}$ is $\dots\dots\dots$
(a) $2 + i$ (b) $2 - i$ (c) $\frac{2-i}{5}$ (d) $\frac{2+i}{5}$
- (5) Which of the following considering factorization of the expression : $X^2 + 4$?
(a) $(X - 2)(X + 2)$ (b) $(X + 2)^2$
(c) $(X - 2i)^2$ (d) $(X - 2i)(X + 2i)$

2 If $7i = (X + 3i)(y - i) - 9$, find the values of the two real numbers X and y which satisfy the previous equation.

3 If a and b are two real numbes , $X = \frac{2+i}{2-i}$, $y = \frac{2+3i}{2+i}$ and $2X - y = a + bi$, prove that : $9a^2 + b^2 = 1$

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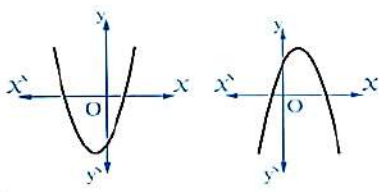
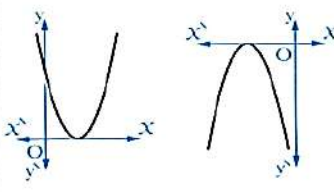
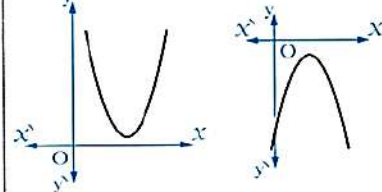
Lesson

2

Determining the types of roots of a quadratic equation

Discriminant

- Using the formula in solving the quadratic equation : $aX^2 + bX + c = 0$, where $a \neq 0$, we get two roots : $\frac{-b + \sqrt{b^2 - 4ac}}{2a}$, $\frac{-b - \sqrt{b^2 - 4ac}}{2a}$
- Both of these two roots include the expression : $\sqrt{b^2 - 4ac}$
- The expression : $b^2 - 4ac$ is called the discriminant of the quadratic equation because it is used to determine the types of roots of the quadratic equation as follows :

Discriminant	positive $(b^2 - 4ac) > 0$	equal to zero $b^2 - 4ac = 0$	negative $(b^2 - 4ac) < 0$
Type of the two roots	Two different real roots	Two equal real roots	Two complex and non real roots
A sketch for the function related to the equation			

Example 1

Determine the type of the two roots of each of the following equations :

1 $X^2 - 3X + 5 = 0$

2 $X^2 + 10X + 25 = 0$

3 $3X^2 + 10X = 4$

Solution

1 $\therefore a = 1$, $b = -3$, $c = 5$

$\therefore$ The discriminant $= b^2 - 4ac = (-3)^2 - 4 \times 1 \times 5 = -11$ (negative quantity)

$\therefore$ The two roots are complex and non real.

2 $\therefore a = 1$, $b = 10$, $c = 25$

$\therefore$ The discriminant $= b^2 - 4ac = (10)^2 - 4 \times 1 \times 25 = 0$

$\therefore$ The two roots are real and equal.

3 $\therefore 3X^2 + 10X - 4 = 0$

$\therefore a = 3$, $b = 10$, $c = -4$

$\therefore$ The discriminant $= b^2 - 4ac = (10)^2 - 4 \times 3 \times (-4) = 148$ (positive quantity)

$\therefore$ The two roots are different and real.

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Example 2

Prove that the two roots of the equation : $7x^2 - 11x + 5 = 0$ are two complex and non real roots , then use the formula to find these two roots.

Solution

$$\therefore a = 7 \quad , \quad b = -11 \quad , \quad c = 5$$

$$\therefore \text{The discriminant} = b^2 - 4ac = (-11)^2 - 4 \times 7 \times 5 = -19 < 0$$

$\therefore$ The two roots are complex and non real roots.

$$\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{11 \pm \sqrt{-19}}{14} = \frac{11 \pm \sqrt{19}i}{14}$$

$$\therefore \text{The two roots of the equation are } \frac{11 + \sqrt{19}i}{14} , \frac{11 - \sqrt{19}i}{14}$$

Example 3

If the two roots of the equation : $x^2 - kx + 2k - 4x + 5 = 0$ are equal , then find the real values of k and find these two roots.

Solution

Put the equation on the general form

$$\therefore x^2 - (k+4)x + (2k+5) = 0$$

$$\therefore \text{The discriminant} = (k+4)^2 - 4 \times 1 \times (2k+5) = k^2 + 8k + 16 - 8k - 20 = k^2 - 4$$

$\therefore$ The two roots of the equation are equal

$$\therefore \text{The discriminant} = 0$$

$$\therefore k^2 - 4 = 0$$

$$\therefore k^2 = 4$$

$$\therefore k = \pm 2$$

$$\therefore \text{at } k = 2 \quad \therefore \text{The equation is } x^2 - 6x + 9 = 0$$

$$\therefore (x-3)^2 = 0$$

$$\therefore x = 3$$

at $k = 2$ the two roots are equal , each one = 3

$$\therefore \text{at } k = -2 \quad \therefore \text{The equation is } x^2 - 2x + 1 = 0$$

$$\therefore (x-1)^2 = 0$$

$$\therefore x = 1$$

at $k = -2$ the two roots are equal , each one = 1

Example 4

1 Find the real values of m which satisfy that the equation : $x^2 - (2m-1)x + m^2 = 0$ has no real roots (**i.e.** has no solutions in $\mathbb{R}$)

2 Find the real values of k which satisfy that the equation : $x^2 + 2(k-1)x + k^2 = 0$ has two real roots (**i.e.** has solutions in $\mathbb{R}$)

Solution

1 $\therefore$ The equation does not have real roots $\therefore b^2 - 4ac < 0$

$$\therefore (2m-1)^2 - 4m^2 < 0$$

$$\therefore 4m^2 - 4m + 1 - 4m^2 < 0$$

$$\therefore -4m < -1$$

$$\therefore m > \frac{1}{4}$$

$$\therefore \text{The equation has no real roots if } m \in \left] \frac{1}{4}, \infty \right[$$

2 $\therefore$ The equation has two real roots

$\therefore$ The two roots are either different or equal

$$\therefore b^2 - 4ac \geq 0$$

$$\therefore 4(k-1)^2 - 4 \times 1 \times k^2 \geq 0$$

$$\therefore 4k^2 - 8k + 4 - 4k^2 \geq 0$$

$$\therefore -8k \geq -4$$

$$\therefore k \leq \frac{1}{2}$$

$$\therefore \text{The equation has two real roots if } k \in \left] -\infty, \frac{1}{2} \right]$$

SIMPLEST MATHS BOOKLETS

Example 5

Prove that for all real values of a , there is no real roots for the equation :
 $4x^2 - 12ax + 9a^2 + 4 = 0$

Solution

The discriminant $= (-12a)^2 - 4(4)(9a^2 + 4)$
 $= 144a^2 - 144a^2 - 64 = -64$ (is negative quantity for all values of a)
 $\therefore$ There is no real roots of the equation.

Remark

If the coefficients a , b and c in the quadratic equation : $ax^2 + bx + c = 0$ are rational numbers and the discriminant is a perfect square, then the roots are real rational numbers.

For example :

1 The equation : $3x^2 - 5x - 2 = 0$

- The terms coefficients are : 3, -5, -2 (rational numbers)
- The discriminant = 49 (perfect square number)
- $\therefore$ The roots are real rational

—— To verify that ——

By substitution in the general formula, the roots are 2, $-\frac{1}{3}$ (real rational)

2 The equation : $x^2 - 2\sqrt{5}x + 1 = 0$

- The terms coefficients are : 1, $-2\sqrt{5}$, 1 (the middle term coefficient is irrational real)
- The discriminant = 16 (perfect square number)
- $\therefore$ The roots are real irrational

—— To verify that ——

By substitution in the general formula, the roots are $\sqrt{5} + 2$, $\sqrt{5} - 2$ (real irrational)

Notice that in the equation $x^2 - 2\sqrt{5}x + 1 = 0$

although the discriminant is perfect square number, the roots are real irrational because the coefficient of the middle term is irrational.

Example 6

If a and b are rational numbers,

prove that the two roots of the equation : $ax^2 + (a^2 + b^2)x + ab^2 = 0$ are rational.

Solution

$$\therefore \text{The discriminant} = (a^2 + b^2)^2 - 4 \times a \times ab^2 = a^4 + 2a^2b^2 + b^4 - 4a^2b^2$$

$$= a^4 - 2a^2b^2 + b^4 = (a^2 - b^2)^2 \text{ is a perfect square}$$

$\therefore$ The coefficients are rational numbers and the discriminant is a perfect square

$\therefore$ The two roots of the equation are rational.

Remark

If the discriminant of the quadratic equation (of real coefficients) isn't positive, then the two roots of the quadratic equation are two conjugate complex numbers.

For example :

The equation $x^2 - 2x + 2 = 0$

- The terms coefficients are : 1, -2, 2 (real numbers)
- The discriminant = -4 (not positive)

$\therefore$ The roots are conjugate complex and to verify that substitute in the general formula the roots are :
 $1 + i$, $1 - i$ (conjugate complex)

SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

- 1 The two roots of the equation : $x^2 - 5x + 11 = 0$ are
 (a) two complex and non real roots. (b) two rational roots.
 (c) two different real roots. (d) two equal real roots.
- 2  The two roots of the equation : $x(x - 2) = 5$ are
 (a) two complex and non real roots. (b) two equal real roots.
 (c) two different real roots. (d) 2 and zero.
- 3 The two roots of the equation : $x + \frac{9}{x} = 6$ are
 (a) two equal real roots. (b) two complex and non real roots.
 (c) two different real roots. (d) two equal imaginary numbers.
- 4 The two roots of the equation : $6x^2 = 19x - 15$ are
 (a) two non real roots. (b) two equal real roots.
 (c) two different rational numbers. (d) two conjugate imaginary numbers.
- 5 Number of values of real x which satisfy the equation : $2x^2 - 7x = 5$ is
 (a) zero (b) 1 (c) 2 (d) 3
- 6 The discriminant of the equation : $(x + 2)^2 + 5 = 0$ is
 (a) perfect square. (b) more than zero.
 (c) negative number. (d) irrational number.
- 7 The quadratic equation : $a^2x^2 + 2abx + b^2 = 0$ where $a \in \mathbb{R}^*$, $b \in \mathbb{R}$
 (a) has two different real roots. (b) has two equal real roots.
 (c) hasn't any real roots. (d) can't determine the type of its two roots.
- 8 The two roots of the equation : $cx^2 + ax + b = 0$ are two complex and non real roots if
 (a) $b^2 - 4ac < 0$ (b) $a^2 - 4bc < 0$
 (c) $c^2 - 4ab < 0$ (d) $b^2 - 4ac > 0$

SIMPLEST MATHS BOOKLETS

9	If the two roots of the equation : $aX^2 + b = 0$ are two different real roots , then (a) $ab > 0$ (b) $a = 0$ (c) $a > 0, b > 0$ (d) $ab < 0$
10	If a $X^2 + bX + c = 0$ and $ac < 0$, then the two roots of the equation are (a) equal real. (b) different real. (c) conjugate complex. (d) rational.
11	If a $X^2 + bX + c = 0$ is a quadratic equation , then which of the following inequalities does satisfy that the equation has two real roots ? (a) $b^2 + 4ac \geq 0$ (b) $b^2 - 4ac < 0$ (c) $b^2 \geq 5ac$ (d) $b^2 - 4ac \leq 0$
12	If a $X^2 + bX + c = 0$ where a, b, c are rational numbers , $a \neq 0$ and $b^2 - 4ac = 25$, then the two roots of the equation are (a) equal real. (b) complex and non real. (c) conjugate complex. (d) different rational.
13	If the two roots of the equation : $X^2 - kX + 25 = 0$ are equal real roots , then $k =$ (a) 10 only. (b) - 10 only. (c) ± 10 (d) ± 5
14	If the two roots of the quadratic equation : $kX^2 - 2kX + 3 = 0$ are equal real roots , then $k =$ (a) zero or 3 (b) ± 1 (c) zero only. (d) 3 only.
15	If the discriminant of the quadratic equation : $2X^2 + 5X + 4k = 0$ equal zero , then k (a) ± 14 (b) zero (c) $\pm \frac{25}{32}$ (d) $\frac{25}{32}$
16	If the two roots of the equation : $X^2 - 4X + k = 0$ are real , then $k \in$ (a) $[4, \infty[$ (b) $] - \infty, 4[$ (c) $]4, \infty[$ (d) $] - \infty, 4]$
17	If the roots of the equation : $X^2 + 3X + k = 0$ are different real , then k can not be (a) - 1 (b) 3 (c) 2 (d) 1

SIMPLEST MATHS BOOKLETS

18

If the roots of the equation : $kx^2 - 8x + 16 = 0$ are two complex and non real , then

- (a) $k > 2$ (b) $k < 2$ (c) $k \in]1, 10[$ (d) $k > 1$

19

In the equation : $75x^2 + 7kx + 3 = 0$ if $k \geq 5$, then the two roots of the equation

- (a) equal real. (b) complex and non real.
(c) different rational. (d) different real.

20

If the graph of the quadratic function $f : f(x)$ does not intersect the x -axis , then which of the following can be the rule of the function ?

- (a) $2x^2 + 3x - 5$ (b) $-x^2 + 5x + 1$
(c) $4x^2 - 20x + 25$ (d) $3x^2 - x + 2$



SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

1 Determine the type of the two roots of each of the following equations :

(1) $x^2 - 2x + 5 = 0$

(2) $x^2 - 10x + 25 = 0$

(3) $-x^2 + 5x - 30 = 0$

(4) $(x - 11) - x(x - 6) = 0$

(5) $x - \frac{2}{x-1} = 4$

(6) $(x - 1)(x - 7) = 2(x - 3)(x - 4)$

2 Prove that : The two roots of the equation : $2x^2 - 3x + 2 = 0$ are complex and not real , then use the general formula to find those two roots.

3 If the two roots of each of the following quadratic equations are equal , then find the value of k :

(1) $x^2 - 3x + 2 + \frac{1}{k} = 0$

« 4 »

(2) $x^2 + (2k + 3)x + k^2 = 0$

« $-\frac{3}{4}$ »

(3) $x^2 + 2(k - 1)x + (2k + 1) = 0$, then find the two roots.

« 0 , 1 , 1 or 4 , -3 , -3 »

(4) $x^2 - 2kx + 7k - 6x + 9 = 0$, then find the two roots.

« 0 , 3 , 3 or 1 , 4 , 4 »

4 Find the values of the real number m that make the equation :

$(m - 1)x^2 - 2mx + m = 0$ has no real roots.

« $m \in]-\infty , 0[$ »

5 If L and M are two rational numbers , then prove that the two roots of the equation : $Lx^2 + (L - M)x - M = 0$ are rational numbers.

6 Prove that for all real values of a and b , the roots of the equation :

$(x - a)(x - b) = 5$ are real.



Higher Skills

1 Choose the correct answer from those given :

(1) The two roots of the equation $x^2 - 2\sqrt{5}x + 1 = 0$ are

(a) real and rational.

(b) not real.

(c) real and equal.

(d) real and irrational.

(2) If $ax^2 + bx + c = 0$, $a \in \mathbb{R}^*$, $b \in \mathbb{R}$, $c \in \mathbb{R}$ and $(b^2 - 4ac)$ is non-positive , then the two roots of the equation are

(a) equal.

(b) not real.

(c) complex and conjugate to each other.

(d) real and different.

2 If a , b and c are real numbers , then prove that the two roots of the equation :

$x^2 + 2ax + a^2 = b^2 + c^2$ are real.

SIMPLEST MATHS BOOKLETS

Lesson

3

Relation between the two roots of the second degree equation and the coefficients of its terms

We know that the two roots of the quadratic equation : $aX^2 + bX + c = 0$, $a \neq 0$ are :

$$\frac{-b + \sqrt{b^2 - 4ac}}{2a} , \quad \frac{-b - \sqrt{b^2 - 4ac}}{2a} , \text{ then :}$$

$$1 \text{ The sum of the two roots} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{-2b}{2a} = \frac{-b}{a}$$

i.e. The sum of the two roots = $\frac{-\text{Coefficient of } X}{\text{Coefficient of } X^2}$

$$2 \text{ The product of the two roots} = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a} = \frac{b^2 - (b^2 - 4ac)}{4a^2}$$

$$= \frac{b^2 - b^2 + 4ac}{4a^2} = \frac{4ac}{4a^2} = \frac{c}{a}$$

i.e. The product of the two roots = $\frac{\text{Absolute term}}{\text{Coefficient of } X^2}$

In a symbolic form , we write :

If L and M are the two roots of the quadratic equation : $aX^2 + bX + c = 0$, then :

$$1 \quad L + M = \frac{-b}{a}$$

$$2 \quad LM = \frac{c}{a}$$

Example 1

Without solving the equation , find the sum and the product of the two roots of each of the following equations :

$$1 \quad 2X^2 + 5X - 12 = 0$$

$$2 \quad 6X^2 - 11X = 10$$

Solution

$$1 \quad \therefore a = 2 , \quad b = 5 , \quad c = -12$$

$$\therefore \text{The sum of the two roots} = \frac{-b}{a} = \frac{-5}{2}$$

$$\therefore \text{the product of the two roots} = \frac{c}{a} = \frac{-12}{2} = -6$$

Check the solution with noticing that the two roots are

$$\frac{3}{2} \text{ and } -4$$

$$2 \quad \therefore 6X^2 - 11X - 10 = 0$$

$$\therefore a = 6 , \quad b = -11 , \quad c = -10$$

$$\therefore \text{The sum of the two roots} = \frac{-b}{a} = \frac{-(-11)}{6} = \frac{11}{6}$$

$$\therefore \text{the product of the two roots} = \frac{c}{a} = \frac{-10}{6} = \frac{-5}{3}$$

SIMPLEST MATHS BOOKLETS

Example 2

- 1 If the sum of the two roots of the equation : $2x^2 + kx + 1 = 0$ is $-\frac{3}{2}$, then find the value of k , and solve the equation in the set of complex numbers.
- 2 If the product of the two roots of the equation : $2x^2 - 4x + k = 0$ is $4\frac{1}{2}$, then find the value of k , and solve the equation in the set of complex numbers.

Solution

- 1 $\therefore$ The sum of the two roots $= -\frac{3}{2}$ $\therefore \frac{-k}{2} = -\frac{3}{2}$
 $\therefore k = 3$ $\therefore$ The equation is $2x^2 + 3x + 1 = 0$
 $\therefore (2x + 1)(x + 1) = 0$ $\therefore x = -\frac{1}{2}$ or $x = -1$
- 2 $\therefore$ The product of the two roots $= 4\frac{1}{2} = \frac{9}{2}$ $\therefore \frac{k}{2} = \frac{9}{2}$ $\therefore k = 9$
 $\therefore$ The equation is $2x^2 - 4x + 9 = 0$ $\therefore a = 2$, $b = -4$, $c = 9$
 $\therefore x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 2 \times 9}}{2 \times 2} = \frac{4 \pm \sqrt{-56}}{4} = \frac{4 \pm \sqrt{56}i}{4} = \frac{4 \pm 2\sqrt{14}i}{4} = 1 \pm \frac{\sqrt{14}}{2}i$
 $\therefore x = 1 + \frac{\sqrt{14}}{2}i$ or $x = 1 - \frac{\sqrt{14}}{2}i$

Example 3

- 1 If $x = -3$ is one of the two roots of the equation : $2x^2 + kx - 3 = 0$, then find the other root, and find the value of k
- 2 If $x = 6$ is one of the two roots of the equation : $x^2 - 5x + k = 0$, then find the other root, and find the value of k
- 3 If -1 and 5 are the two roots of the equation : $ax^2 + bx - 5 = 0$, then find the value of each of a and b

Solution

- 1 $\therefore$ The product of the two roots $= \frac{c}{a} = -\frac{3}{2}$ $\therefore -3 \times \text{the other root} = -\frac{3}{2}$
 $\therefore$ The other root $= \frac{-3}{2} \times \frac{1}{-3}$ $\therefore$ The other root $= \frac{1}{2}$
 $\therefore$ The sum of the two roots $= -\frac{b}{a} = -\frac{k}{2}$, $\therefore -3 + \frac{1}{2} = -\frac{k}{2}$
 $\therefore$ The two roots are -3 , $\frac{1}{2}$ $\therefore k = 5$
 $\therefore \frac{-5}{2} = -\frac{k}{2}$

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Example 4

If $(1 + \sqrt{2}i)$ is one of the two roots of the equation : $x^2 - 2x + c = 0$ where $c \in \mathbb{R}$, then find :

1 The other root.

2 The value of c

Solution

$$\therefore \text{The sum of the two roots} = \frac{-(-2)}{1} = 2$$

$$\therefore (1 + \sqrt{2}i) + \text{the other root} = 2$$

$$\therefore \text{The other root} = 2 - (1 + \sqrt{2}i)$$

i.e. The other root $= 1 - \sqrt{2}i$

$$\therefore \text{The product of the two roots} = c$$

$$\therefore 1^2 - (\sqrt{2}i)^2 = c$$

$$\therefore 1 + 2 = c$$

$$\therefore (1 - \sqrt{2}i)(1 + \sqrt{2}i) = c$$

$$\therefore 1 - 2i^2 = c$$

$$\therefore c = 3$$

Notice that

$\therefore$ Coefficients of the terms are real and one of the two roots is non real complex number

$\therefore$ The other root is the conjugate of the given root.

i.e. It equals $(1 - \sqrt{2}i)$

Remarks

In the quadratic equation : $ax^2 + bx + c = 0$

1 If $a = 1$, then $L + M = -b$ and $LM = c$

i.e. The sum of the two roots = the additive inverse of the coefficient of x ,
the product of the two roots = the absolute term.

2 If $b = 0$, then $L + M = 0$, **i.e.** $L = -M$

i.e. One of the two roots of the equation is the additive inverse of the other.

3 If $a = c$, then $LM = 1$, **i.e.** $L = \frac{1}{M}$

i.e. One of the two roots of the equation is the multiplicative inverse of the other.

Example 5

1 Find the value of k , if one of the roots of the equation : $3x^2 + (k - 3)x + 7 = 0$ is the additive inverse of the other root.

2 Find the value of k , if one of the roots of the equation : $2kx^2 + 7x + k^2 + 1 = 0$ is the multiplicative inverse of the other.

Solution

1 $\therefore$ One of the roots is the additive inverse of the other

$$\therefore b = 0$$

$$\therefore k - 3 = 0$$

$$\therefore k = 3$$

2 $\therefore$ One of the roots is the multiplicative inverse of the other

$$\therefore a = c$$

$$\therefore k^2 + 1 = 2k$$

$$\therefore k^2 - 2k + 1 = 0$$

$$\therefore (k - 1)^2 = 0$$

$$\therefore k = 1$$

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Example 6

Find the value of d , if one of the two roots of the equation : $x^2 + d x - 50 = 0$ is double the additive inverse of the other root.

Solution

Let one of the two roots = L

$\therefore$ The other root = $-2 L$

$\therefore$ the product of the two roots = $\frac{\text{absolute term}}{\text{coefficient of } x^2}$

$$\therefore L(-2 L) = \frac{-50}{1}$$

$$\therefore -2 L^2 = -50$$

$$\therefore L^2 = 25$$

$$\therefore L = \pm 5$$

$\therefore$ the sum of the two roots = $\frac{-\text{coefficient of } x}{\text{coefficient of } x^2}$

$$\therefore L + (-2 L) = \frac{-d}{1}$$

$$\therefore -L = -d$$

$$\therefore L = d$$

$$\therefore d = \pm 5$$

Example 7

Find the satisfying condition which makes one of the two roots of the equation : $a x^2 + b x + c = 0$ equal to the additive inverse of twice the other root.

Solution

Let one of the two roots be L

$\therefore$ The other root = $-2 L$

$\therefore$ the sum of the two roots = $\frac{-b}{a}$

$$\therefore L + (-2 L) = \frac{-b}{a}$$

$$\therefore L = \frac{b}{a} \quad (1)$$

$\therefore$ The product of the two roots = $\frac{c}{a}$

$$\therefore L \times (-2 L) = \frac{c}{a}$$

$$\therefore L^2 = \frac{-c}{2a} \quad (2)$$

By substituting from (1) in (2) :

$$\therefore \left(\frac{b}{a}\right)^2 = \frac{-c}{2a}$$

$$\therefore \frac{b^2}{a^2} = \frac{-c}{2a}$$

$$\therefore \frac{b^2}{a} = \frac{-c}{2}$$

$$\therefore 2 b^2 + a c = 0 \quad (\text{That is the required condition})$$

SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

- 1 The sum of the two roots of the equation : $x^2 + 3x - 10 = 0$ is
(a) 10 (b) - 10 (c) 3 (d) - 3
- 2 The sum of the two roots of the equation : $5x^2 - 3 = 0$ is
(a) $\frac{3}{5}$ (b) $-\frac{3}{5}$ (c) zero (d) $\frac{5}{3}$
- 3 The product of the two roots of the equation : $2x^2 - 7x - 6 = 0$ equal
(a) - 6 (b) $\frac{7}{2}$ (c) 3 (d) - 3
- 4 The product of the two roots of the equation : $3 + 2x - \frac{1}{4}x^2 = 0$ equals
(a) $-\frac{2}{3}$ (b) 12 (c) - 12 (d) $\frac{3}{4}$
- 5 The product of the two roots of the equation : $3x^2 - 4 = 0$ multiplying by the sum of the two roots of the equation $x^2 - 3x = 0$ is
(a) 12 (b) - 3 (c) - 4 (d) 3
- 6 If the sum of the two roots of the equation : $3x^2 + bx + 14 = 0$ is $-\frac{7}{3}$, then b =
(a) - 7 (b) 7 (c) $\frac{14}{3}$ (d) - 14
- 7 If the product of the two roots of the equation : $(k - 2)x^2 - 6x + 12 = 0$ is 3 , then k =
(a) zero (b) 4 (c) 6 (d) 38
- 8 If M , (5 - M) are the two roots of the equation : $x^2 - kx + 6 = 0$, then k =
(a) - 5 (b) 5 (c) 6 (d) - 8
- 9 In the quadratic equation : $bx^2 + cx + a = 0$, if the sum of the two roots equal the product of them , then c =
(a) b (b) a (c) - b (d) - a
- 10 If $x = -1$ is one of the two roots of the equation : $x^2 - kx - 6 = 0$, then the sum of the two roots =
(a) - 5 (b) 6 (c) - 6 (d) 5

SIMPLEST MATHS BOOKLETS

11 If $(2 - i)$ is one of the roots of the equation : $X^2 + bX + c = 0$ where $b, c \in \mathbb{R}$, then $(b, c) = \dots\dots\dots$

- (a) $(4, 5)$ (b) $(-4, -5)$ (c) $(4, -5)$ (d) $(-4, 5)$

12 If L, M are the two roots of the equation : $X^2 - (k + 2)X - 3 = 0$ and $L + M = 0$, then $k = \dots\dots\dots$

- (a) -2 (b) -3 (c) 2 (d) 3

13 If $M, \frac{2}{M}$ are the roots of the equation : $aX^2 + bX + 12 = 0$, then $a = \dots\dots\dots$

- (a) 3 (b) 5 (c) 6 (d) 9

14 If $(L + 1), (M + 1)$ are the two roots of the equation : $X^2 - 3X + 2 = 0$ and $L < M$, then $L = \dots\dots\dots$

- (a) zero (b) 1 (c) 2 (d) 3

15 If L, M are the two roots of the equation : $X^2 + X + 1 = 0$, then $L + M + LM = \dots\dots\dots$

- (a) zero (b) 1 (c) -1 (d) 2

16 If L, M are the two roots of the equation : $X^2 - 21X + 4 = 0$, then : $\sqrt{L} + \sqrt{M} = \dots\dots\dots$

- (a) 25 (b) 5 (c) -5 (d) ± 5

17 If the two roots of the equation : $X^2 + bX + c = 0$ are L and L , then $b^2 + 4c = \dots\dots\dots$

- (a) 0 (b) $4L^2$ (c) $8L$ (d) $8L^2$

18 The product of the roots of the equations : $aX^2 + bX + c = 0$, $bX^2 + cX + a = 0$ and $cX^2 + aX + b = 0$ equal $\dots\dots\dots$ (where a, b and c are non-zero real numbers)

- (a) abc (b) -1 (c) 1 (d) zero

19  If one of the two roots of the equation : $X^2 - (b - 3)X + 5 = 0$ is the additive inverse of the other root, then $b = \dots\dots\dots$

- (a) -5 (b) -3 (c) 3 (d) 5

20  If one of the two roots of the equation : $aX^2 - 3X + 2 = 0$ is the multiplicative inverse of the other, then $a = \dots\dots\dots$

- (a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 2 (d) 3

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Ex.2) Answer the following:

1 Without solving the equation , find the sum and the product of the two roots of each of the following equations :

(1) $3x^2 = 23x - 30$

(2) $(4x + 1)(x + 6) = (x - 2)(3x - 4)$

(3) $\frac{x}{2} + \frac{1}{x} = \frac{3}{2}$

(4) $(a - 1)x^2 + x - a^2x - 1 + a = 0$

2 If the product of the two roots of the equation : $3x^2 + 10x - c = 0$ is $-\frac{8}{3}$, find the value of c , then solve the equation in the set of complex numbers. « $c = 8$, $x = \frac{2}{3}$ or $x = -4$ »

3 If the sum of the two roots of the equation : $2x^2 + bx - 5 = 0$ is $-\frac{3}{2}$, find the value of b , then solve the equation in the set of complex numbers. « $b = 3$, $x = \frac{-5}{2}$ or $x = 1$ »

4 Find the other root of the equation , then find the value of a in each of the following where $a \in \mathbb{R}$:

(1) If $x = -1$ is one of the two roots of the equation : $x^2 - 2x + a = 0$ « 3 , -3 »

(2) If $(1 + i)$ is one of the two roots of the equation : $x^2 - 2x + a = 0$ « $1 - i$, 2 »

5 Find the values of a , b in each of the following equations , if :

(1) 2 , 5 are the two roots of the equation : $x^2 + ax + b = 0$ « $a = -7$, $b = 10$ »

(2) -3 , 7 are the two roots of the equation : $ax^2 - bx - 21 = 0$ « $a = 1$, $b = 4$ »

(3) -1 , $\frac{3}{2}$ are the two roots of the equation : $ax^2 - x + b = 0$ « $a = 2$, $b = -3$ »

(4) $\sqrt{3}i$, $-\sqrt{3}i$ are the two roots of the equation : $x^2 + ax + b = 0$ « $a = 0$, $b = 3$ »

6 Find the value of k in each of the following which makes :

(1) One of the roots of the equation : $x^2 + (k - 1)x - 3 = 0$ is the additive inverse of the other roots. « 1 »

(2) One of the roots of the equation : $4kx^2 + 7x + k^2 + 4 = 0$ is the multiplicative inverse of the other. « 2 »

Higher Skills

1 Choose the correct answer from those given :

(1) If (2 i) is one root of the equation : $x^2 + ax + b = 0$

where coefficients of its terms are real numbers , then all of the following are true except

(a) the other root is $(-2i)$

(b) sum of the two roots = zero

(c) product of the two roots = -4

(d) discriminant of the equation < 0

(2) To evaluate the real values of b , c in the equation : $x^2 + bx + c = 0$, it is sufficient to have

(a) real roots sum = 6 only.

(b) one of the roots = $(3 + i)$ only.

(c) (a) , (b) together.

(d) nothing of the previous.

2 Find the value of a which makes the two roots of the equation :

$3x^2 - (2a - 1)x + (a - 4) = 0$ are different in sign.

« $a \in]-\infty , 4[$ »

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Lesson

4

Forming the quadratic equation whose two roots are known

Let L and M be the two roots of the quadratic equation : $aX^2 + bX + c = 0$

By multiplying the two sides by $\frac{1}{a}$ where $a \neq 0$, the equation becomes in the form :

$$X^2 + \frac{b}{a}X + \frac{c}{a} = 0 \quad \text{i.e.} \quad X^2 - \left(\frac{-b}{a}\right)X + \frac{c}{a} = 0 \quad (1)$$

But $\frac{-b}{a} = L + M$, $\frac{c}{a} = LM$

By substituting in (1) , we get the quadratic equation whose roots are L , M which is :

$$X^2 - (L + M)X + LM = 0 \quad (2)$$

i.e. $X^2 - (\text{the sum of the two roots})X + \text{the product of the two roots} = 0$

And by factorizing the trinomial in the left side of the equation (2) , we get another form of the last equation which is $(X - L)(X - M) = 0$

Example 1

Form the quadratic equation whose roots are :

1 $\frac{3}{2}, \frac{5}{4}$

2 $3 + \sqrt{2}, 3 - \sqrt{2}$

3 $\frac{-1+i}{i}, \frac{2}{1+i}$

Solution

- 1 The sum of the two roots $= \frac{3}{2} + \frac{5}{4} = \frac{11}{4}$, the product of them $= \frac{3}{2} \times \frac{5}{4} = \frac{15}{8}$
 $\therefore$ the equation is $X^2 - (\text{the sum of the two roots})X + \text{the product of the two roots} = 0$
 $\therefore$ The equation is $X^2 - \frac{11}{4}X + \frac{15}{8} = 0$ (by multiplying by 8)
 $\therefore$ The equation is $8X^2 - 22X + 15 = 0$

Forming a quadratic equation from the roots of another equation

Example 2

If the two roots of the equation : $X^2 - 5X - 6 = 0$ are L , M , find the equation whose roots are $L + 7$, $M + 7$

Solution

The required in this example is forming an equation using a given equation where there is a certain relation between the roots of the two equations. There are many methods for solving this example and we will mention them in the following :

The first method

- Find the two roots of the given equation.
- Find the two roots of the required equation.
- Form the required equation.

$$\therefore X^2 - 5X - 6 = 0$$

$$\therefore (X - 6)(X + 1) = 0$$

$\therefore 6, -1$ are the two roots of the given equation.

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Let $L = 6$, $M = -1$, the two roots of the required equation be D , E

$$\therefore D = L + 7 = 6 + 7 = 13 \text{ , } E = M + 7 = -1 + 7 = 6$$

$$\therefore D + E = 13 + 6 = 19 \text{ , } DE = 13 \times 6 = 78$$

$$\therefore \text{The required equation is } X^2 - 19X + 78 = 0$$

The second method

Let D and E be the two roots of the required equation

$$\therefore D = L + 7 \text{ , } E = M + 7$$

$$\therefore D + E = L + 7 + M + 7 = L + M + 14$$

$$\therefore L + M = 5 \text{ (from the given equation)}$$

$$\therefore D + E = 5 + 14 = 19$$

$$\therefore DE = (L + 7)(M + 7) = LM + 7(L + M) + 49$$

$$\therefore LM = -6 \text{ (from the given equation)}$$

$$\therefore DE = -6 + 7 \times 5 + 49 = 78$$

$$\therefore \text{The required equation is } X^2 - 19X + 78 = 0$$

The third method

Let D and E be the two roots of the required equation

$$\therefore D = L + 7 \text{ , } E = M + 7$$

$$\therefore L = D - 7 \text{ , } M = E - 7$$

$$\therefore L \text{ is one of the two roots of the given equation : } X^2 - 5X - 6 = 0$$

$$\therefore L^2 - 5L - 6 = 0$$

$$\therefore L = D - 7$$

$$\therefore (D - 7)^2 - 5(D - 7) - 6 = 0$$

$$\therefore D^2 - 14D + 49 - 5D + 35 - 6 = 0$$

$$\therefore D^2 - 19D + 78 = 0$$

i.e. D is a root of the equation : $X^2 - 19X + 78 = 0$ (which is the required equation)

Remark

The third method is used only if the relation between the first root of the given equation and the first root of the required equation is the same relation between the second root of the given equation and the second root of the required equation.

Remember the following identities

$$1 \quad L^2 + M^2 = (L + M)^2 - 2LM$$

$$3 \quad L^3 + M^3 = (L + M) [(L + M)^2 - 3LM]$$

$$5 \quad \frac{1}{M} + \frac{1}{L} = \frac{L + M}{LM}$$

$$2 \quad (L - M)^2 = (L + M)^2 - 4LM$$

$$4 \quad L^3 - M^3 = (L - M) [(L + M)^2 - LM]$$

$$6 \quad \frac{L}{M} + \frac{M}{L} = \frac{L^2 + M^2}{LM} = \frac{(L + M)^2 - 2LM}{LM}$$

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Example 3

If L, M are the two roots of the equation : $x^2 - 7x + 9 = 0$ where $L > M$, find the numerical value of each of the following expressions :

- | | |
|---------------|---------------------|
| 1 $L^2 + M^2$ | 2 $L^2 + 3LM + M^2$ |
| 3 $L - M$ | 4 $L^3 - M^3$ |

Solution

$\therefore L, M$ are the two roots of the equation : $x^2 - 7x + 9 = 0 \quad \therefore L + M = 7$ and $LM = 9$

$$1 \quad L^2 + M^2 = (L + M)^2 - 2LM = (7)^2 - 2 \times 9 = 49 - 18 = 31$$

$$2 \quad L^2 + 3LM + M^2 = (L^2 + 2LM + M^2) + LM = (L + M)^2 + LM = (7)^2 + 9 = 49 + 9 = 58$$

$$3 \quad (L - M)^2 = (L + M)^2 - 4LM = (7)^2 - 4 \times 9 = 49 - 36 = 13$$

$$\therefore L - M = \sqrt{13}, \text{ where } L > M$$

$$4 \quad L^3 - M^3 = (L - M) [(L + M)^2 - LM]$$

by substituting from (3) :

$$\therefore L^3 - M^3 = \sqrt{13} (7^2 - 9) = \sqrt{13} (49 - 9) = 40\sqrt{13}$$

Example 4

If the two roots of the equation : $x^2 - 8x + 5 = 0$ are L and M

, form the equation whose roots are $\frac{1}{L}$ and $\frac{1}{M}$

Solution

$\therefore L$ and M are the two roots of the given equation. $\therefore L + M = 8$ and $LM = 5$

$\therefore \frac{1}{L}$ and $\frac{1}{M}$ are the two roots of the required equation.

$$\therefore \text{The sum of the two roots} = \frac{1}{L} + \frac{1}{M} = \frac{M + L}{LM} = \frac{8}{5}$$

$$\therefore \text{the product of the two roots} = \frac{1}{L} \times \frac{1}{M} = \frac{1}{LM} = \frac{1}{5}$$

$$\therefore \text{The required equation is } x^2 - \frac{8}{5}x + \frac{1}{5} = 0$$

$$\text{i.e. } 5x^2 - 8x + 1 = 0$$

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Example 5

If L and M are the two roots of the equation :

$x^2 - 5x + 9 = 0$, find the equation whose roots are L^2 and M^2

Solution

$\therefore$ L and M are the two roots of the given equation. $\therefore L + M = 5$ and $LM = 9$

$\therefore L^2$ and M^2 are the two roots of the required equation.

$\therefore$ The sum of the two roots $= L^2 + M^2 = (L + M)^2 - 2LM = 5^2 - 2 \times 9 = 7$

, the product of the two roots $= L^2 \times M^2 = (LM)^2 = 9^2 = 81$

$\therefore$ The required equation is $x^2 - 7x + 81 = 0$

Example 6

If L and M are the two roots of the equation :

$3x^2 + 5x - 7 = 0$, find the equation whose roots are $L + \frac{1}{M}$, $M + \frac{1}{L}$

Solution

$\therefore$ L and M are the two roots of the given equation.

$\therefore L + M = -\frac{5}{3}$ and $LM = -\frac{7}{3}$

, $\therefore L + \frac{1}{M}$, $M + \frac{1}{L}$ are the two roots of the required equation.

$\therefore$ The sum of the two roots $= L + \frac{1}{M} + M + \frac{1}{L} = L + M + \frac{L+M}{LM}$
 $= -\frac{5}{3} + \frac{-\frac{5}{3}}{-\frac{7}{3}} = -\frac{5}{3} + \frac{5}{7} = \frac{-35 + 15}{21} = -\frac{20}{21}$

, the product of the two roots $= \left(L + \frac{1}{M}\right) \left(M + \frac{1}{L}\right) = LM + \frac{1}{LM} + 2$
 $= -\frac{7}{3} - \frac{3}{7} + 2 = \frac{-49 - 9 + 42}{21} = -\frac{16}{21}$

$\therefore$ The required equation is $x^2 - \frac{-20}{21}x + \frac{-16}{21} = 0$

i.e. $21x^2 + 20x - 16 = 0$

SIMPLEST MATHS BOOKLETS

Example 7

If $\frac{2}{L}$, $\frac{2}{M}$ are the two roots of the equation : $x^2 - 6x + 4 = 0$,

find the equation whose roots are L, M

Solution

$\therefore \frac{2}{L}$, $\frac{2}{M}$ are the two roots of the given equation.

$$\therefore \frac{2}{L} \times \frac{2}{M} = 4$$

$$\therefore \frac{4}{LM} = 4$$

$$\therefore LM = 1$$

$$\therefore \frac{2}{L} + \frac{2}{M} = 6$$

$$\therefore \frac{2L + 2M}{LM} = 6$$

$$\therefore \frac{2(L + M)}{1} = 6$$

$$\therefore L + M = \frac{6}{2} = 3$$

$\therefore$ L and M are the two roots of the required equation, $L + M = 3$, $LM = 1$

$\therefore$ The required equation is $x^2 - 3x + 1 = 0$

Example 8

If the difference between the two roots of the equation : $x^2 - kx + 4k = 0$ equals three times the product of the two roots of the equation : $x^2 - 3x - k = 0$, find the value of k

Solution

Let L and M be the two roots of the equation : $x^2 - kx + 4k = 0$

$$\therefore L + M = k, \quad LM = 4k$$

$\therefore$ the difference between L and M equals three times the product of the two roots of the equation : $x^2 - 3x - k = 0$

$$\therefore L - M = -3k$$

$$\therefore (L - M)^2 = (L + M)^2 - 4LM \text{ (from the previous identities)}$$

$$\therefore (-3k)^2 = k^2 - 4(4k)$$

$$\therefore 9k^2 = k^2 - 16k$$

$$\therefore 8k^2 + 16k = 0$$

$$\therefore 8k(k + 2) = 0$$

$$\therefore k = 0 \text{ or } k + 2 = 0$$

$$\therefore k = -2$$

Another solution :

By using the law of the difference between the two roots :

$$\therefore L - M = \frac{\pm \sqrt{\text{the discriminant}}}{a} = \frac{\pm \sqrt{b^2 - 4ac}}{a} \text{ and from the equation :}$$

$x^2 - kx + 4k = 0$, we found that :

$$L - M = \pm \sqrt{k^2 - 16k} \quad (1)$$

$\therefore$ L - M equals three times the product

of the two roots of : $x^2 - 3x - k = 0$

$$\therefore L - M = -3k \quad (2)$$

$\therefore$ from (1), (2) :

$$\therefore \pm \sqrt{k^2 - 16k} = -3k, \text{ by squaring both sides}$$

$$\therefore k^2 - 16k = 9k^2$$

$$\therefore 8k^2 + 16k = 0$$

$$\therefore k = 0 \text{ or } k = -2$$

Remark

It is possible to deduce the law of the difference between the two roots from the general formula with the same method used for finding the sum of the two roots in the previous lesson.

SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

- 1 The quadratic equation whose roots sum equals -1 and their product equals -3 is
 (a) $x^2 - x - 3 = 0$ (b) $x^2 + x + 3 = 0$
 (c) $x^2 - x + 3 = 0$ (d) $x^2 + x - 3 = 0$
- 2 The quadratic equation whose roots are $-2, 3$ is
 (a) $(x + 2)(x + 3) = 0$ (b) $x^2 - 4x + 6 = 0$
 (c) $x^2 - x = 6$ (d) $4x^2 - 2x + 3 = 0$
- 3 The quadratic equation whose roots are $-2i$ and $2i$ is
 (a) $x^2 = 4i$ (b) $x^2 + 4 = 0$ (c) $x^2 - 4 = 0$ (d) $ix^2 + 4 = 0$
- 4 The quadratic equation whose roots are $\frac{3}{2}i$ and $\frac{3}{2}i^3$ is
 (a) $4x^2 - 9 = 0$ (b) $4x^2 + 9 = 0$ (c) $4x^2 - 4 = 0$ (d) $9x^2 + 4 = 0$
- 5 The quadratic equation whose roots are $(1 - 5i)$ and $(1 + 5i)$ is
 (a) $x^2 - 2x + 26 = 0$ (b) $x^2 + 2x - 26 = 0$
 (c) $x^2 - 2x - 26 = 0$ (d) $x^2 + 2x + 26 = 0$
- 6 If L, M are the two roots of the equation : $x^2 - 4x + 1 = 0$, then the value of expression : $L^2 - 4L + 1 =$
 (a) zero (b) -4 (c) 1 (d) -1
- 7 If L is one of the roots of the equation : $x^2 + 4x + 7 = 0$, then $(L + 2)^2 =$
 (a) -11 (b) 11 (c) 3 (d) -3
- 8 If L, M are the two roots of the equation : $x^2 - 7x + 3 = 0$, then the value of the expression : $L^2 M + L M^2 =$
 (a) 7 (b) 3 (c) 10 (d) 21
- 9 If L, M are the two roots of the equation : $x^2 - 7x + 3 = 0$, then $L^2 + M^2 =$
 (a) 7 (b) 43 (c) 58 (d) 79

SIMPLEST MATHS BOOKLETS

10	If L , M are the two roots of the equation : $x^2 - 8x + c = 0$ and $L^2 + M^2 = 40$, then c =
	(a) 8 (b) 10 (c) 12 (d) 14
11	If L , M are the two roots of the equation : $x^2 - 7x + 9 = 0$ where $L > M$, then $L^3 - M^3 = \dots\dots\dots$
	(a) 31 (b) 63 (c) $40\sqrt{13}$ (d) $9\sqrt{7}$
12	If L , M are the two roots of the equation : $x^2 - 5x + 7 = 0$, then $L(M + 1) + M = \dots\dots\dots$
	(a) 2 (b) - 2 (c) 12 (d) 7
13	If L , M are the two roots of the equation : $3x^2 - 8x + 2 = 0$, then $\frac{1}{L} + \frac{1}{M} = \dots\dots\dots$
	(a) $\frac{4}{3}$ (b) 4 (c) $\frac{-4}{3}$ (d) $\frac{2}{3}$
14	If L , M are the two roots of the equation : $x^2 - 7x + 3 = 0$, then the equation whose two roots are (L + M) and LM is
	(a) $x^2 - 10x + 21 = 0$ (b) $x^2 + 10x + 21 = 0$ (c) $x^2 - 21x + 10 = 0$ (d) $x^2 - 21x - 10 = 0$
15	If L , M are the two roots of the equation : $x^2 - 5x + 3 = 0$, then the equation whose two roots are 2L , 2M is
	(a) $2x^2 - 10x + 6 = 0$ (b) $x^2 - 10x + 12 = 0$ (c) $2x^2 - 10x - 6 = 0$ (d) $x^2 + 10x + 12 = 0$
16	If L , M are the two roots of the equation : $2x^2 - 3x - 6 = 0$, then the equation whose two roots are $\frac{L}{4}$ and $\frac{M}{4}$ is
	(a) $x^2 - 3x - 6 = 0$ (b) $4x^2 - 6x - 3 = 0$ (c) $16x^2 + 6x - 3 = 0$ (d) $16x^2 - 6x - 3 = 0$
17	If L , M are the two roots of the equation : $x^2 - 5x + 7 = 0$, then the equation whose two roots are L^2 and M^2 is
	(a) $x^2 + 11x + 49 = 0$ (b) $x^2 - 11x + 49 = 0$ (c) $x^2 - 49x + 11 = 0$ (d) $x^2 + 11x - 49 = 0$

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Ex.2) Answer the following:

- 1** Form the quadratic equation whose two roots are :
- | | | |
|-----------------------------------|--------------------------------------|---|
| (1) $-2, 4$ | (2) $7, 7$ | (3) $-7, 0$ |
| (4) $\frac{2}{3}, \frac{3}{2}$ | (5) $\frac{3}{5}, -2\frac{1}{5}$ | (6) $5\sqrt{3}, -2\sqrt{3}$ |
| (7) $7+2\sqrt{5}, 7-2\sqrt{5}$ | (8) $-5i, 5i$ | (9) $1-3i, 1+3i$ |
| (10) $3-2\sqrt{2}i, 3+2\sqrt{2}i$ | (11) $\frac{3}{i}, \frac{3+3i}{1-i}$ | (12) $\frac{-2+2i}{1+i}, \frac{-2-4i}{2-i}$ |
- 2** If L and M are the two roots of the equation : $x^2 - 7x + 5 = 0$, then find the numerical value of each of the following expressions :
- | | | |
|---------------------|--|---|
| (1) $L^2 M + M^2 L$ | (2) $\frac{1}{M} + \frac{1}{L}$ | |
| (3) $(L-2)(M-2)$ | (4) $(L + \frac{1}{M})(M + \frac{1}{L})$ | $\ll 35, \frac{7}{5}, -5, 7\frac{1}{5} \gg$ |
- 3** If L and M are the two roots of the equation : $x^2 - 3x - 5 = 0$, then find the equation whose roots are : $L-4$ and $M-4$ $\ll x^2 + 5x - 1 = 0 \gg$
- 4** Find the quadratic equation in which each of the two roots exceeds one of the two roots of the equation : $x^2 - 7x - 9 = 0$ $\ll x^2 - 9x - 1 = 0 \gg$
- 5** Find the quadratic equation in which each of its two roots equals the square of the corresponding root of the equation : $x^2 + 3x - 5 = 0$ $\ll x^2 - 19x + 25 = 0 \gg$
- 6** If L and M are the two roots of the equation : $2x^2 - 3x - 1 = 0$, then form the quadratic equations whose two roots are : $\frac{L}{M}, \frac{M}{L}$ $\ll 2x^2 + 13x + 2 = 0 \gg$
- 7** If the difference between the two roots of the equation : $x^2 + kx + 2k = 0$ equals twice the product of the two roots of the equation : $x^2 + 3x + k = 0$, then find the value of k $\ll 0 \text{ or } -\frac{8}{3} \gg$

Third Higher skills

Choose the correct answer from those given :

- (1) The quadratic equation whose roots are the dimensions of a rectangle of area 15 cm^2 and its perimeter 26 cm . is
- | | |
|--------------------------|--------------------------|
| (a) $x^2 - 26x + 15 = 0$ | (b) $x^2 + 26x - 15 = 0$ |
| (c) $x^2 - 13x - 15 = 0$ | (d) $x^2 - 13x + 15 = 0$ |
- (2) If $a^2 + 3a + 1 = 0$, $b^2 + 3b + 1 = 0$ where a , b are real different numbers , then $\frac{a}{b} + \frac{b}{a} = \dots\dots\dots$
- | | | | |
|-------|-------|--------|--------|
| (a) 2 | (b) 7 | (c) -5 | (d) 11 |
|-------|-------|--------|--------|
- (3) If L , M are the roots of the quadratic equation : $(x-a)(x-b) = k$, then the quadratic equation whose roots are a and b is
- | | |
|----------------------|----------------------------|
| (a) $(x-L)(x-M) = 0$ | (b) $(x-L)(x-M) + k = 0$ |
| (c) $(x-L)(x-M) = k$ | (d) $x^2 - (L+M)x + k = 0$ |

SIMPLEST MATHS BOOKLETS

Lesson

5

Sign of a function

Investigating the sign of a function

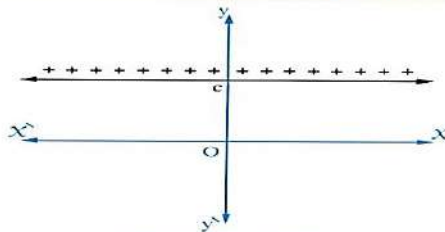
Investigating the sign of a function f in the variable X is to determine the values of X at which the values of the function f are as follows :

- Positive , **i.e.** $f(X) > 0$
- Negative , **i.e.** $f(X) < 0$
- Equal to zero , **i.e.** $f(X) = 0$

First The sign of the constant function

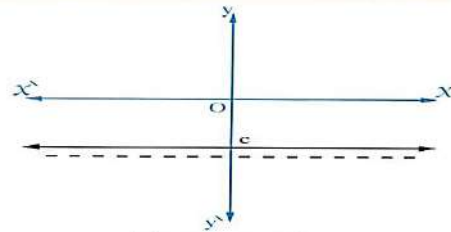
The following figures represent the two functions :

$f : f(X) = c$ (where c is **positive**)



We notice that ———
The function is positive for all $X \in \mathbb{R}$

$f : f(X) = c$ (where c is **negative**)



We notice that ———
The function is negative for all $X \in \mathbb{R}$

From the previous , we deduce that :

The sign of the constant function $f : f(X) = c$, $c \in \mathbb{R}^*$ is the same sign of $c \forall X \in \mathbb{R}$

Notice that

The symbol $\forall$ means "for every"

1a

For example :

- If $f(X) = 5$, then the sign of the function f is positive for all $X \in \mathbb{R}$
- If $f(X) = -3$, then the sign of the function f is negative for all $X \in \mathbb{R}$

TRY TO SOLVE

Determine the sign of each of the following two functions :

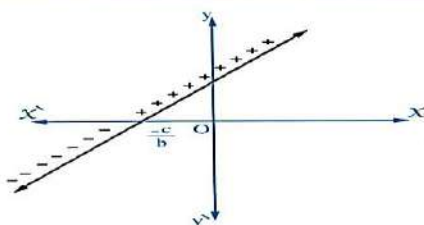
1 $f : f(X) = 10$

2 $f : f(X) = -\frac{2}{5}$

Second The sign of the first degree function (linear function)

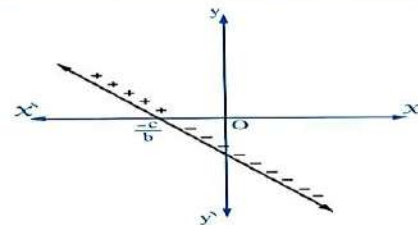
The following figures represent the two functions :

$f : f(X) = bX + c$ (b is **positive**)



- We notice that the sign of the function ,**
- is the same as the sign of b (**positive**) at $X > -\frac{c}{b}$
 - is opposite to the sign of b (**negative**) at $X < -\frac{c}{b}$
 - equals **zero** at $X = -\frac{c}{b}$

$f : f(X) = bX + c$ (b is **negative**)



- We notice that the sign of the function ,**
- is the same as the sign of b (**negative**) at $X > -\frac{c}{b}$
 - is opposite to the sign of b (**positive**) at $X < -\frac{c}{b}$
 - equals **zero** at $X = -\frac{c}{b}$

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From the previous , we deduce that :

To find the sign of the linear function $f : f(x) = b x + c$, $b \neq 0$, we put $f(x) = 0$

$$\therefore b x + c = 0 \quad \therefore x = \frac{-c}{b}$$

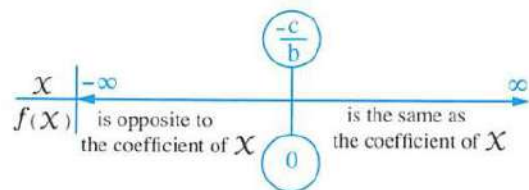
$\therefore$ The sign of the function f :

1 Is the same as the sign of b at $x > \frac{-c}{b}$

2 Is opposite to the sign of b at $x < \frac{-c}{b}$

3 $f(x) = 0$ at $x = \frac{-c}{b}$

And we illustrate this on the opposite number line.



Example 1

Determine the sign of each of the following two functions using the number line :

1 $f : f(x) = 3x + 6$

2 $f : f(x) = 1 - \frac{1}{2}x$

Solution

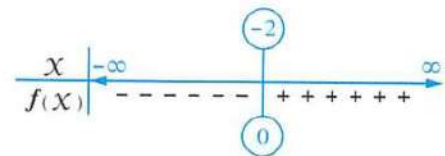
1 $\therefore f(x) = 3x + 6$ put $f(x) = 0$

$$\therefore 3x + 6 = 0 \quad \therefore x = -2$$

$\therefore$ The sign of the function f is :

- positive at $x > -2$
- negative at $x < -2$
- $f(x) = 0$ at $x = -2$

We illustrate the solution on the opposite number line.



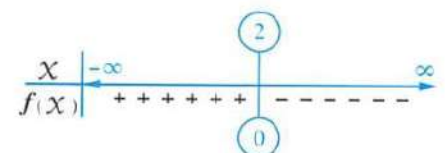
2 $\therefore f(x) = -\frac{1}{2}x + 1$ put $f(x) = 0$

$$\therefore -\frac{1}{2}x = -1 \quad \therefore x = 2$$

$\therefore$ The sign of the function f is :

- negative at $x > 2$
- positive at $x < 2$
- $f(x) = 0$ at $x = 2$

We illustrate the solution on the opposite number line.



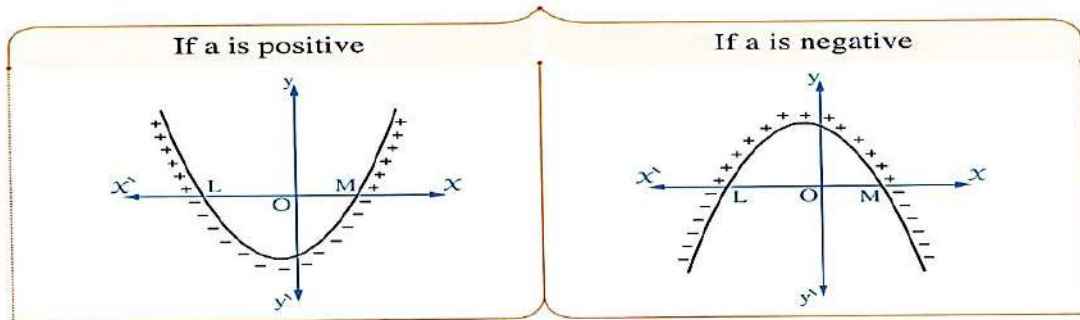
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Third The sign of the second degree function (quadratic function)

To determine the sign of the quadratic function $f : f(x) = ax^2 + bx + c$, $a \neq 0$, we have to obtain the discriminant of the equation : $ax^2 + bx + c = 0$, there are three cases :

1 The discriminant : $b^2 - 4ac > 0$

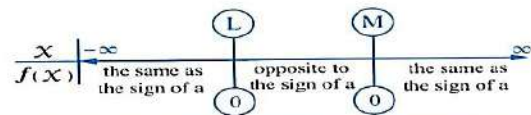
The equation has two real roots, let them be L , M where $L < M$



The sign of the function is as follows :

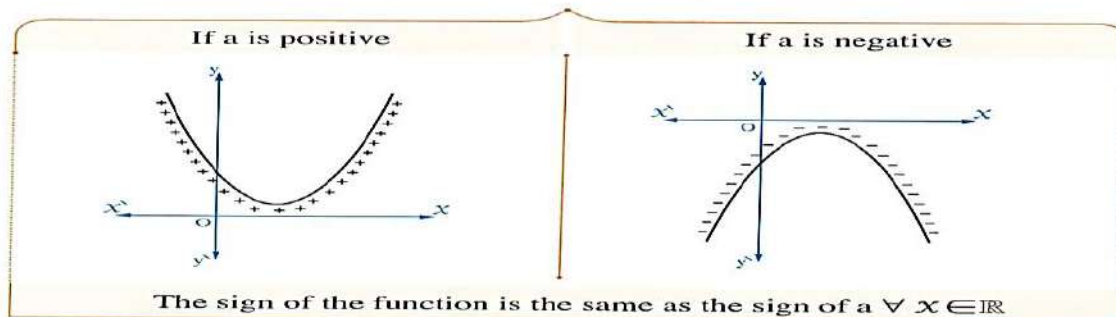
- Is the same as the sign of a at $x \in \mathbb{R} - [L, M]$
- Is opposite to the sign of a at $x \in]L, M[$
- Equals zero at $x \in \{L, M\}$

And we illustrate this on the opposite number line.



2 The discriminant : $b^2 - 4ac < 0$

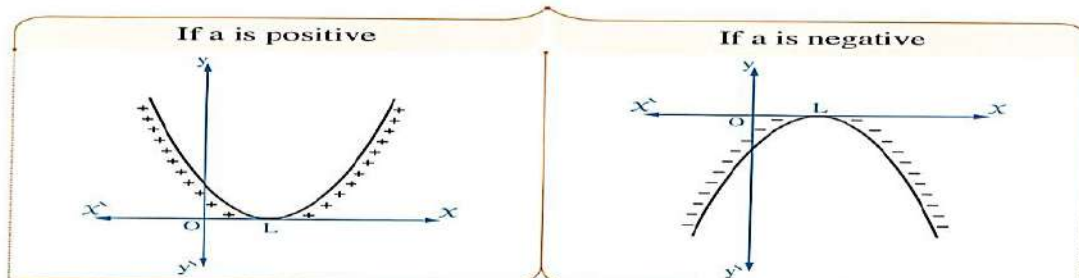
There is no real roots for the equation and thus the sign of the function is as follows :



The sign of the function is the same as the sign of $a \forall x \in \mathbb{R}$

3 The discriminant : $b^2 - 4ac = 0$

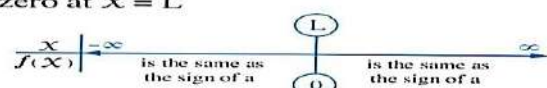
There are two equal roots for the equation, let each of them be L



The sign of the function is as follows :

- Is the same as a at $x \neq L$
- Is equal to zero at $x = L$

We can illustrate this on the opposite number line.



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Example 2

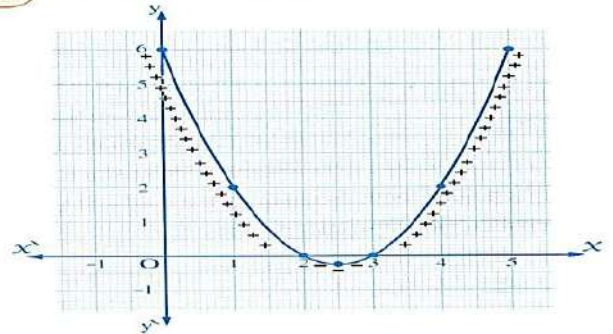
Draw the graph of the function : $f : f(x) = x^2 - 5x + 6$ in the interval $[0, 5]$, from the graph determine the sign of the function f in $\mathbb{R}$.

Solution

x	0	1	2	2.5	3	4	5
$f(x)$	6	2	0	-0.25	0	2	6

From the graph, we notice that the sign of f is :

- Positive at $x \in \mathbb{R} - [2, 3]$
- Negative at $x \in]2, 3[$
- $f(x) = 0$ at $x \in \{2, 3\}$



Remark

If the required is investigating the sign of the function in the given interval, then the sign of f is :

- Positive at $x \in [0, 2] \cup]3, 5]$ or $[0, 5] -]2, 3[$
- Negative at $x \in]2, 3[$
- $f(x) = 0$ at $x \in \{2, 3\}$

Remember that

In the previous example :

- The domain of the function f is the set of the real numbers $\mathbb{R}$
- The range of the function f is $[-0.25, \infty[$
- The vertex of the curve is $(2.5, -0.25)$ and the function has a minimum value equals -0.25
- The symmetry axis equation is $x = 2.5$

Example 3

Draw the graph of the function :

$f : f(x) = -x^2 + 4x - 4$ in the interval $[0, 4]$

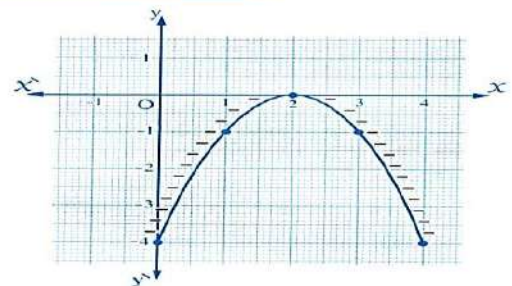
, from the graph determine the sign of the function f in $\mathbb{R}$

Solution

x	0	1	2	3	4
$f(x)$	-4	-1	0	-1	-4

From the graph, we notice that :

- $f(x) = 0$ at $x = 2$
- The sign of f is negative at $x \in \mathbb{R} - \{2\}$



Example 4

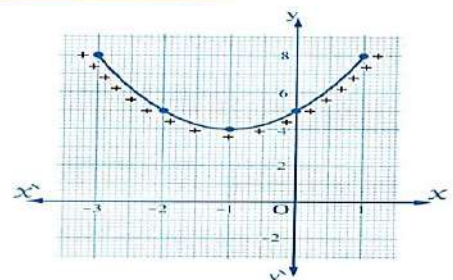
Draw the graph of the function :

$f : f(x) = x^2 + 2x + 5$ in the interval $[-3, 1]$

, from the graph determine the sign of the function f in $\mathbb{R}$

x	-3	-2	-1	0	1
$f(x)$	8	5	4	5	8

From the graph, we notice that the sign of the function f is positive $\forall x \in \mathbb{R}$



SIMPLEST MATHS BOOKLETS

Example 5

Determine the sign of each of the following functions, showing that on the number line :

1 $f : f(x) = x^2 + 2x - 3$

2 $f : f(x) = x^2 - 3x + 5$

3 $f : f(x) = 4x^2 - 12x + 9$

4 $f : f(x) = 9 + 2x - x^2$

Solution

1 $\therefore$ The discriminant $= b^2 - 4ac = 4 - 4 \times 1 \times (-3) = 4 + 12 = 16 (> \text{zero})$

$\therefore$ The equation $x^2 + 2x - 3 = 0$ has two roots.

By factorization $\therefore (x + 3)(x - 1) = 0$

$\therefore x = -3$ or $x = 1$

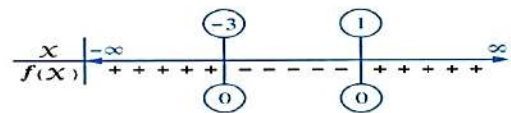
$\therefore a$ (coefficient of x^2) $= 1 > 0$

$\therefore$ The sign of the function f is :

• positive at $x \in \mathbb{R} - [-3, 1]$

• negative at $x \in]-3, 1[$

• $f(x) = 0$ at $x \in \{-3, 1\}$



2 $\therefore$ The discriminant $= b^2 - 4ac = 9 - 4 \times 1 \times 5 = 9 - 20 = -11 (< \text{zero})$

$\therefore$ The equation : $x^2 - 3x + 5 = 0$ has no real roots

$\therefore a = 1 > 0$

$\therefore$ The sign of the function f is positive $\forall x \in \mathbb{R}$



3 $\therefore$ The discriminant $= b^2 - 4ac = 144 - 4 \times 4 \times 9 = 144 - 144 = 0$

$\therefore$ The equation : $4x^2 - 12x + 9 = 0$ has two equal roots

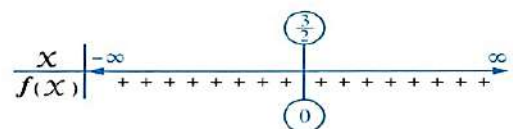
By factorization : $\therefore (2x - 3)^2 = 0 \quad \therefore x = \frac{3}{2}$

$\therefore a = 4 > 0$

$\therefore$ The sign of the function f is :

• positive at $x \in \mathbb{R} - \left\{ \frac{3}{2} \right\}$

• $f(x) = 0$ at $x = \frac{3}{2}$



4 $\therefore$ The discriminant $= b^2 - 4ac = 4 - 4 \times (-1) \times 9 = 40 (> \text{zero})$

$\therefore$ The equation : $9 + 2x - x^2 = 0$ has two roots.

By using the general formula

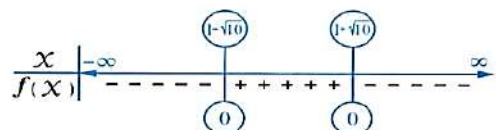
$$\therefore x = \frac{-2 \pm \sqrt{40}}{-2} = \frac{-2 \pm 2\sqrt{10}}{-2} = 1 \pm \sqrt{10}$$

$\therefore a$ (coefficient of x^2) $= -1 < 0 \quad \therefore$ The sign of the function f is :

• negative at $x \in \mathbb{R} - [1 - \sqrt{10}, 1 + \sqrt{10}]$

• positive at $x \in]1 - \sqrt{10}, 1 + \sqrt{10}[$

• $f(x) = 0$ at $x \in \{1 - \sqrt{10}, 1 + \sqrt{10}\}$



SIMPLEST MATHS BOOKLETS

Example 6

If $f : f(x) = x - 1$, $g : g(x) = x^2 + x - 6$

, find the interval at which the two functions f , g are positive together , also the interval at which f , g are negative together.

Solution

$$\therefore f(x) = x - 1$$

, f is positive at $x > 1$

, f is negative at $x < 1$

$$\therefore f(x) = 0 \text{ at } x = 1$$

i.e. In the interval $]1, \infty[$

i.e. In the interval $]-\infty, 1[$

$$\therefore g(x) = x^2 + x - 6 ,$$

We get the two roots of the equation $x^2 + x - 6 = 0$ as follows :

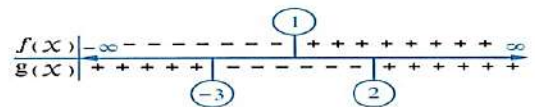
$$(x - 2)(x + 3) = 0 \quad \therefore x = 2 \text{ or } x = -3$$

$$\therefore g(x) = 0 \text{ at } x \in \{2, -3\}$$

, g is positive at $x \in \mathbb{R} - [-3, 2]$

, g is negative at $x \in]-3, 2[$

By noticing the opposite figure we find :



- f , g are positive together in the interval

$]2, \infty[$ which is the interval representing $]1, \infty[\cap \mathbb{R} - [-3, 2]$

- f , g are negative together at $]-3, 1[$ which is equal to $]-\infty, 1[\cap]-3, 2[$

Example 7

Prove that for all values of $x \in \mathbb{R}$ the two roots of the equation : $x^2 + 2kx + k - 2 = 0$ are real and different.

Solution

$$\therefore x^2 + 2kx + k - 2 = 0$$

$$\therefore a = 1 , b = 2k , c = k - 2$$

$$\therefore \text{The discriminant} = b^2 - 4ac = (2k)^2 - 4(k - 2) = 4k^2 - 4k + 8$$

and the two roots are real and different if the discriminant is positive ,

thus we investigate the sign of the function

$$f : f(k) = 4k^2 - 4k + 8 \text{ as follows :}$$

$$\therefore \text{The discriminant} = b^2 - 4ac = (-4)^2 - 4 \times 4 \times 8 = 16 - 128 = -112 (< \text{zero})$$

$$\therefore \text{The equation } 4k^2 - 4k + 8 = 0 \text{ has no real roots , } \therefore a > 0$$

$\therefore$ The sign of the function f is positive for all the values of $k \in \mathbb{R}$

$\therefore$ The discriminant of the equation $x^2 + 2kx + k - 2 = 0$ is positive $\forall x \in \mathbb{R}$

Thus the two roots of the equation $x^2 + 2kx + k - 2 = 0$ are real and different $\forall x \in \mathbb{R}$

Another solution :

$$\therefore \text{The discriminant of the equation : } x^2 + 2kx + k - 2 = 0 \text{ is } 4k^2 - 4k + 8$$

$$\therefore 4k^2 - 4k + 8 = 4k^2 - 4k + 1 + 7 = (2k - 1)^2 + 7 \text{ is positive } \forall k \in \mathbb{R}$$

$$\therefore \text{The two roots of the equation } x^2 + 2kx + k - 2 = 0 \text{ are real and different } \forall x \in \mathbb{R}$$

SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

- 1 The function $f : f(x) = -4$ is negative in the interval
 (a) $]-\infty, 4[$ only. (b) $]-4, 4[$ only.
 (c) $]-\infty, \infty[$ (d) $]-2, 2[$ only.
- 2 The function $f : f(x) = 5x - 3$ is positive at
 (a) $x > \frac{3}{5}$ (b) $x < \frac{3}{5}$ (c) $x > \frac{1}{3}$ (d) $x < \frac{-5}{3}$
- 3 If $f(x) = 2x - 4$, then f is negative at $x \in$
 (a) $[2, \infty[$ (b) $]-\infty, 2[$ (c) $[2, \infty[$ (d) $]-\infty, 2]$
- 4 The sign of the function $f : f(x) = 6 - 2x$ is non positive at
 (a) $x > 3$ (b) $x \leq 3$ (c) $x < 3$ (d) $x \geq 3$
- 5 The function $f : f(x) = 3 - \frac{1}{2}x$ is non negative at $x \in$
 (a) $]-\infty, 6]$ (b) $]-\infty, 6[$ (c) $[6, \infty[$ (d) $[6, \infty[$
- 6 If the function $f :]-4, 3[\longrightarrow \mathbb{R}$ where $f(x) = x + 2$, then $f(x)$ is positive at $x \in$
 (a) $]-\infty, -2[$ (b) $]-2, \infty[$ (c) $]-4, -2[$ (d) $]-2, 3]$
- 7 If the function $f :]-5, 6[\longrightarrow \mathbb{R}$ where $f(x) = x + 3$, then $f(x)$ is negative at $x \in$
 (a) $]-5, -3[$ (b) $]-\infty, -3[$ (c) $]-3, \infty[$ (d) $]-3, 6]$
- 8 The function $f : f(x) = c$ has a sign always.
 (a) positive (b) negative
 (c) like the sign of x (d) like the sign of c
- 9 The sign of the function $f : f(x) = ax + b$ on $\mathbb{R}$ is the same as the sign of b if
 (a) $a = b$ (b) $a = 0$ (c) $a > 0$ (d) $a < 0$

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10	The function $f : f(x) = ax^2 + bx + c$ has one sign on $\mathbb{R}$ if (a) $b^2 - 4ac > 0$ (b) $b^2 - 4ac < 0$ (c) $b^2 - 4ac = 0$ (d) $b^2 - 4ac \geq 0$
11	If $f(x) = 3x$, then the sign of the function f is negative in the interval (a) $]-\infty, 3[$ (b) $]3, \infty[$ (c) $]-\infty, 0[$ (d) $]-3, \infty[$
12	The function $f : f(x) = x^2 - 9$ is negative at $x \in$ (a) $\mathbb{R} - [-3, 3]$ (b) $]-3, 3[$ (c) $]-\infty, -9[$ (d) $]-\infty, -3[$
13	The function $f : f(x) = x^2 + 1$ is positive at $x \in$ (a) $]0, \infty[$ only. (b) $]1, \infty[$ only. (c) $]-\infty, 1[$ only. (d) $\mathbb{R}$
14	The function $f : f(x) = x^2 - 6x + 9$ is positive in the interval (a) $]0, \infty[$ (b) $]-\infty, 3]$ (c) $\mathbb{R} - \{3\}$ (d) $\mathbb{R} - \{0\}$
15	The interval in which the function $f : f(x) = x^2 - 5x + 6$ is positive is (a) $[2, 3]$ (b) $\mathbb{R} - \{2, 3\}$ (c) $\mathbb{R} - [2, 3]$ (d) $\mathbb{R} -]2, 3[$
16	If $f(x)$ is positive at $x \in]-2, 5[$, then $f(x) =$ (a) $x^2 - 3x - 10$ (b) $10 - 3x - x^2$ (c) $x^2 + 3x - 10$ (d) $10 + 3x - x^2$
17	If $f(x) = x^2 + bx + c$ is negative at $x \in]2, 3[$ only, then the product of the two roots of the equation : $x^2 + bx + c = 0$ equal (a) -6 (b) 6 (c) b (d) $-c$
18	The sign of the two functions $f : f(x) = (x - 1)(x + 2)$ and $g : g(x) = -x^2 + 9$ are both positive at $x \in$ (a) $]1, 3[\cup]-3, -2[$ (b) $]-2, 0[$ (c) $]3, \infty[\cup]-\infty, -3[$ (d) $]-3, 3[$

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Ex.2) Answer the following:

1 Determine the sign of the functions which are defined by the following rules , then represent your answer on the number line :

(1) $f(x) = (x-2)(x+3)$

(2) $f(x) = (2x-3)^2$

(3) $f(x) = 2x^2 + 5x - 7$

(4) $f(x) = x^2 - 8x + 16$

(5) $f(x) = 2x^2 - 3x + 5$

(6) $f(x) = 2x^2$

2 Draw the curve of the function $f : f(x) = 2x^2 - 3x + 4$ in $[-1, 2\frac{1}{2}]$
From the graph , determine the sign of f in $\mathbb{R}$

3 Draw the curve of the function $f : f(x) = -x^2 + 8x - 15$ in $[1, 7]$ From the graph ,
determine the sign of f in $\mathbb{R}$ and the solution of the equation $f(x) = 0$ « $\{3, 5\}$ »

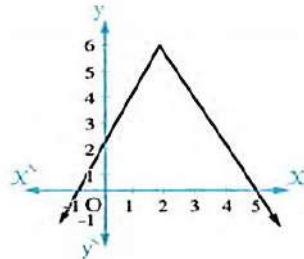
4 Draw the curve of the function $f : f(x) = x^2 - 9$ in the interval $[-3, 4]$
From the graph , determine the sign of f in that interval.

5 Draw the curve of the function $f : f(x) = -x^2 + 2x + 4$ in $[-3, 5]$
From the graph , determine the sign of f in that interval.

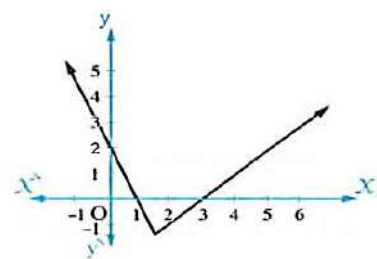
6 Prove that for all the values of $k \in \mathbb{R}$ the two roots of the equation :
 $2x^2 - kx + k - 3 = 0$ are real and different.

7 Determine the sign of the functions represented by the following figures :

(1)



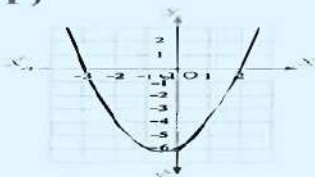
(2)



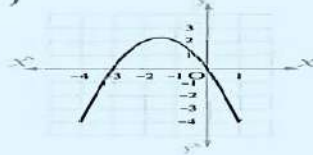
Third Higher skills

Each of the following figures shows the graphical representation of a second degree function in one variable. Study the sign of each function in $\mathbb{R}$, then find the rule of each function :

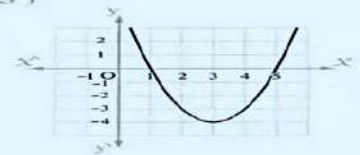
(1)



(2)



(3)



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Lesson

6

Quadratic inequalities in one variable

Solving the quadratic inequalities in $\mathbb{R}$

To solve the quadratic inequality in $\mathbb{R}$, follow the following steps :

- 1 Write the quadratic function related to the inequality.
- 2 Study the sign of this quadratic function.
- 3 Determine the intervals which satisfy the inequality.

Example 1

Find in $\mathbb{R}$ the solution set of the inequality : $x^2 - 5x + 6 > 0$

Solution

First : Write the quadratic function related to the inequality as follows :

$$f(x) = x^2 - 5x + 6$$

Second : Study the sign of f as follows :

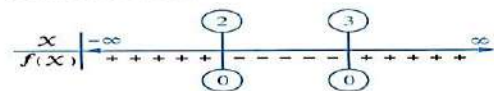
$\therefore$ The discriminant $= b^2 - 4ac = 25 - 4 \times 1 \times 6 = 1 (> \text{zero})$

$\therefore$ The equation : $x^2 - 5x + 6 = 0$ has two different roots.

By factorizing :

$$\therefore (x - 2)(x - 3) = 0$$

$$\therefore x = 2 \quad \text{or} \quad x = 3$$



Third : Determine the intervals which satisfy $x^2 - 5x + 6 > 0$ (positive)

$\therefore$ The solution set $=]-\infty, 2[\cup]3, +\infty[$ or $\mathbb{R} - [2, 3]$



Notice that

From the previous example :

The solution set of the inequality : $x^2 - 5x + 6 < 0$ in $\mathbb{R}$ is $]2, 3[$

Example 2

Find in $\mathbb{R}$ the solution set of the inequality : $(x + 5)(x - 1) \geq x + 5$

Solution

$$\therefore (x + 5)(x - 1) \geq x + 5 \quad \therefore x^2 + 4x - 5 \geq x + 5 \quad \therefore x^2 + 3x - 10 \geq 0$$

First : Write the quadratic function related to the inequality as follows :

$$f(x) = x^2 + 3x - 10$$

Second : Study the sign of the function f as follows :

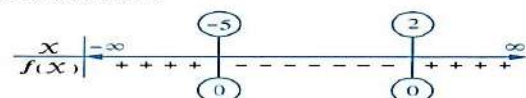
$\therefore$ The discriminant $= b^2 - 4ac = 9 - 4 \times 1 \times (-10) = 49 (> \text{zero})$

$\therefore$ The equation $x^2 + 3x - 10 = 0$ has two different roots

By factorizing :

$$\therefore (x - 2)(x + 5) = 0$$

$$\therefore x = 2 \quad \text{or} \quad x = -5$$



Third : Determine the intervals which satisfy that : $x^2 + 3x - 10 \geq 0$

$\therefore$ The solution set $=$

$$]-\infty, -5] \cup [2, +\infty[\quad \text{or} \quad \mathbb{R} -]-5, 2[$$



Notice that

From the previous example :

The solution set of the inequality : $(x + 5)(x - 1) \leq x + 5$ in $\mathbb{R}$ is $]-5, 2[$

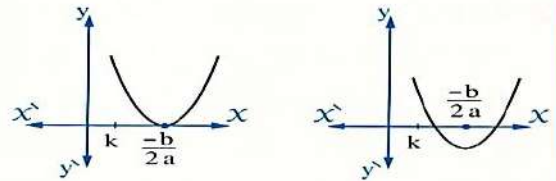
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Remarks

If the quadratic equation $aX^2 + bX + c = 0$ where f is the related function with it, then :

① Conditions that each of the two roots of the equation is greater than a real number k are :

- $b^2 - 4ac \geq 0$
- $a f(k) > 0$
- $-\frac{b}{2a} > k$



For example :

If each of the two roots of the equation $X^2 - 5X + m = 0$ is greater than 2, then :

- $25 - 4m \geq 0 \quad \therefore m \leq 6 \frac{1}{4}$
 - $4 - 5(2) + m > 0 \quad \therefore m > 6$
 - $\frac{5}{2} > 2$ "satisfied for all values of m "
- , then to satisfy the 3 conditions : $6 < m \leq 6 \frac{1}{4}$

② Conditions that only one of the two roots of the equation lies between the two real numbers m, n is : $f(m) \times f(n) < 0$

For example :

If only one root of the equation $X^2 - bX + 12 = 0$ is belong to the interval $]1, 4[$

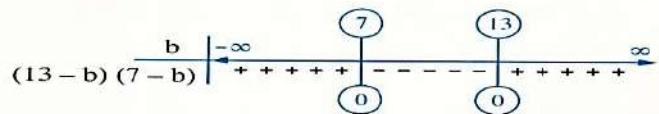
, then $f(1) \times f(4) < 0$

$$\therefore (1 - b + 12)(16 - 4b + 12) < 0$$

$$\therefore (13 - b)(28 - 4b) < 0$$

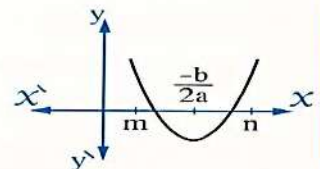
$$\therefore (13 - b)(7 - b) < 0$$

$$\therefore b \in]7, 13[$$



③ Conditions that the two roots of the equation are lying between the two real numbers m, n where $m < n$ are :

- $b^2 - 4ac \geq 0$
- $a f(m) > 0$
- $a f(n) > 0$
- $m < -\frac{b}{2a} < n$



For example :

If the two roots of the equation $4X^2 - 2X + h = 0$ are elements of the interval $] -1, 1[$, then :

- $4 - 4 \times 4 \times h \geq 0 \quad \therefore h \leq \frac{1}{4} \quad \text{①}$
- $4 f(-1) > 0 \quad \therefore 4 \times (4 + 2 + h) > 0 \quad \therefore h > -6 \quad \text{②}$
- $4 f(1) > 0 \quad \therefore 4(4 - 2 + h) > 0 \quad \therefore h > -2 \quad \text{③}$
- $-1 < \frac{2}{2 \times 4} < 1$ satisfies for all values of $h \quad \text{④}$

From ①, ②, ③ and ④ $\therefore -2 \leq h \leq \frac{1}{4}$

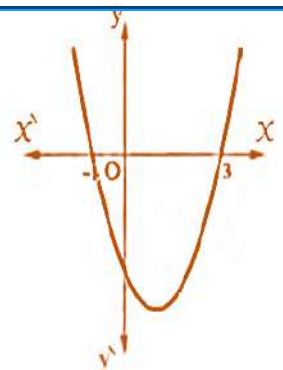
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Ex.1) Choose the correct answer :

- 1 The solution set of the inequality : $(x - 2)(x - 5) < 0$ in $\mathbb{R}$ is
 (a) $\{2, 5\}$ (b) $]2, 5[$ (c) $[2, 5]$ (d) $\mathbb{R} - [2, 5]$
- 2 The solution set of the inequality : $x^2 + 3x - 4 \geq 0$ in $\mathbb{R}$ is
 (a) $\{-4, 1\}$ (b) $[-4, 1]$ (c) $\mathbb{R} -]-4, 1[$ (d) $\mathbb{R} - [-4, 1]$
- 3 The solution set of the inequality : $7 + x^2 - 4x < 0$ in $\mathbb{R}$ is
 (a) $] -4, 7[$ (b) $\mathbb{R} - [-4, 7]$ (c) $\mathbb{R}$ (d) $\emptyset$
- 4 The solution set of the inequality : $2x + x^2 + 5 > 0$ in $\mathbb{R}$ is
 (a) $\mathbb{R} - [-2, 3]$ (b) $[-2, 3]$ (c) $\mathbb{R}$ (d) $\emptyset$
- 5 The solution set of the inequality : $x^2 + 9 > 6x$ in $\mathbb{R}$ is
 (a) $] -3, 3[$ (b) $\mathbb{R}$ (c) $\mathbb{R} - [-3, 3]$ (d) $\mathbb{R} - \{3\}$
- 6 The solution set of the inequality : $4x - x^2 - 4 < 0$ in $\mathbb{R}$ is
 (a) $\mathbb{R}$ (b) $\mathbb{R}^+$ (c) $\mathbb{R}^-$ (d) $\mathbb{R} - \{2\}$
- 7 The S.S. of the inequality $(x - 1)^2 \leq 0$ in $\mathbb{R}$ is
 (a) $\mathbb{R}$ (b) $\emptyset$ (c) $\{1\}$ (d) $\mathbb{R} - \{1\}$
- 8 The solution set of the inequality : $-x(x + 2) \geq 0$ in $\mathbb{R}$ is
 (a) $\{0, -2\}$ (b) $[-2, 0]$ (c) $] -2, 0[$ (d) $[-2, 2]$
- 9 The solution set of the inequality : $x(x - 1) > 0$ in $\mathbb{R}$ is
 (a) $\{0, 1\}$ (b) $]0, 1[$ (c) $[0, 1]$ (d) $\mathbb{R} - [0, 1]$
- 10 The solution set of the inequality : $x(x - 2) < 0$ is
 (a) $\{0, 2\}$ (b) $] -2, 2[$ (c) $]0, 2[$ (d) $]1, 2[$

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11	The solution set of the inequality : $x^2 < 3x$ is
	(a) $\mathbb{R} - [0, 3]$ (b) $[0, 3]$ (c) $]0, 3[$ (d) $\mathbb{R} -]0, 3[$
12	The solution set of the inequality : $x^2 + 1 \leq 0$ in $\mathbb{R}$ is
	(a) $\emptyset$ (b) $\mathbb{R}$ (c) $[-1, 1]$ (d) $\mathbb{R} -]-1, 1[$
13	The solution set of the inequality : $x^2 + 9 > 0$ in $\mathbb{R}$ is
	(a) $\emptyset$ (b) $\mathbb{R}$ (c) $] -3, 3[$ (d) $\mathbb{R} - [-3, 3]$
14	If $f(x) = x^2 - 6x + 9$, then the solution set of the inequality : $f(x) \leq 0$ in $\mathbb{R}$ is
	(a) $\mathbb{R}$ (b) $\{3\}$ (c) $\mathbb{R} -]-3, 3[$ (d) $[-3, 3]$
15	The solution set of the inequality : $x^2 \leq 9$ in $\mathbb{R}^+$ is
	(a) $[-3, 3]$ (b) $\mathbb{R} -]-3, 3[$ (c) $]0, 3[$ (d) $\emptyset$
16	The solution set of the inequality : $x^2 > 16$ in the interval $[-4, 4]$ is
	(a) $[-4, 4]$ (b) $\mathbb{R} - [-4, 4]$ (c) $\emptyset$ (d) $\{-4, 4\}$
17	Which of the following answers does not belong to the solution set of the inequality $3x - 5 \geq 4x - 3$?
	(a) -1 (b) -2 (c) -3 (d) -5
18	If the opposite figure represents the function $f : f(x) = x^2 - 2x - 3$, then the solution set of the inequality $x^2 - 2x - 3 \geq 0$ in $\mathbb{R}$ is
	(a) $]-1, 3[$ (b) $]-\infty, 2[$ (c) $]3, \infty[$ (d) $]-\infty, -1] \cup [3, \infty[$



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Ex.2) Answer the following:

1

Find in $\mathbb{R}$ the solution set of each of the following inequalities :

- | | | |
|--|--|---|
| (1)  $x^2 + 2x - 8 > 0$ | (2) $x^2 - 5x - 6 < 0$ | (3) $4 - 3x - x^2 \geq 0$ |
| (4)  $x^2 - 1 \leq 0$ | (5)  $2x - x^2 < 0$ | (6)  $x^2 - 4x + 4 \geq 0$ |

2

Find in $\mathbb{R}$ the solution set of each of the following inequalities :

- | | |
|---|---|
| (1)  $5 - 2x \leq x^2$ | (2)  $5x^2 + 12x \geq 44$ |
| (3)  $3x^2 \leq 11x + 4$ | (4)  $x^2 \geq 6x - 9$ |
| (5)  $x(x+2) - 3 \leq 0$ | (6)  $x^2 + 5 \leq 1$ |
| (7)  $(x-2)^2 \leq -5$ | (8)  $(x+3)^2 < 10 - 3(x+3)$ |

3

Determine the sign of the function $f : f(x) = x^2 - 5x + 6$ and from that find in $\mathbb{R}$ the solution set of the inequality : $f(x) < 0$

Third

Higher Skills

Choose the correct answer from those given :

- (1) If $f(x) = x^2 - 7x + 12$, $x \in \mathbb{R}$, then all the following are true except
 - (a) solution set of the equation $f(x) = 0$ is $\{3, 4\}$
 - (b) solution set of the inequality $f(x) > 0$ is $\mathbb{R} - [3, 4]$
 - (c) solution set of the inequality $f(x) < 0$ is $]3, 4[$
 - (d) $f(x)$ is positive in the interval $\mathbb{R} -]3, 4[$
- (2) The sum of integers belong to the solution set of the inequality $(x-2)(3x-1) \leq 0$
 - (a) -1 (b) 1 (c) 2 (d) 3
- (3) The solution set of the inequality $(x+3)^2 < 4(x+1)^2$ in $\mathbb{R}$ is
 - (a) $] \frac{-5}{3}, 1[$ (b) $\mathbb{R} -] \frac{-5}{3}, 1[$ (c) $[\frac{-5}{2}, 1]$ (d) $\mathbb{R} - [\frac{-5}{3}, 1]$
- (4) If L, M are the roots of the equation : $ax^2 + bx + c = 0$ where $a > 0$, $L < M$, then the solution set of the inequality $ax^2 + bx + c < 0$ in $\mathbb{R}$ is
 - (a) $] -\infty, L[$ (b) $] L, M[$ (c) $] M, \infty[$ (d) $\mathbb{R} - [L, M]$

Unit 2



TRIGONOMETRY

1

Directed angle

2

**Systems of measuring angle
(Degree measure - Radian measure)**

3

Trigonometric functions

4

Related angles

5

**Graphing trigonometric
functions**

6

**Finding the measure of
an angle given the value
of one of its trigonometric**



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Lesson

1

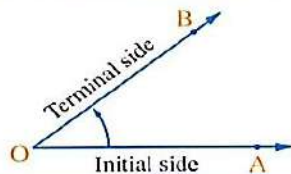
Directed angle

Definition of the directed angle

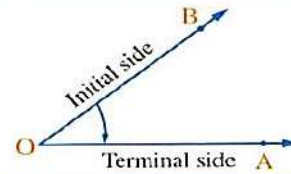
The directed angle is an ordered pair of two rays called the sides of the angle with a common starting point called the vertex.

If $\overrightarrow{OA}$, $\overrightarrow{OB}$ are the two sides of an angle whose vertex is "O", then :

The ordered pair $(\overrightarrow{OA}, \overrightarrow{OB})$ represents the directed angle $\angle AOB$, whose initial side is $\overrightarrow{OA}$, and terminal side is $\overrightarrow{OB}$



The ordered pair $(\overrightarrow{OB}, \overrightarrow{OA})$ represents the directed angle $\angle BOA$ whose initial side is $\overrightarrow{OB}$, and terminal side is $\overrightarrow{OA}$



From the previous, we deduce that :

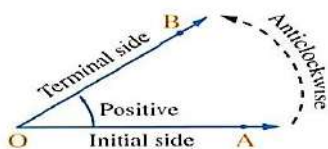
directed angle $\angle AOB \neq$ directed angle $\angle BOA$ because $(\overrightarrow{OA}, \overrightarrow{OB}) \neq (\overrightarrow{OB}, \overrightarrow{OA})$

Positive and negative measures of a directed angle

The measure of the directed angle is

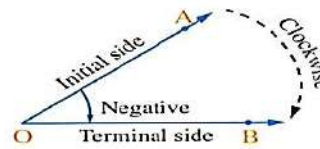
Positive

If the direction of the rotation from the initial side to the terminal side is *anticlockwise*



Negative

If the direction of the rotation from the initial side to the terminal side is *clockwise*

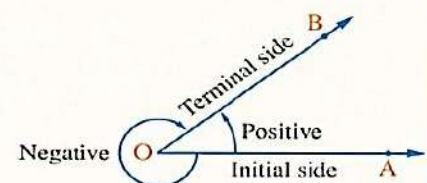


Remark

Each non zero directed angle has two measures, one is positive and the other is negative such that the sum of the absolute values of the two measures equals 360°

i.e. $| \text{Positive measure of the directed angle} |$

$+ | \text{Negative measure of the same directed angle} | = 360^\circ$

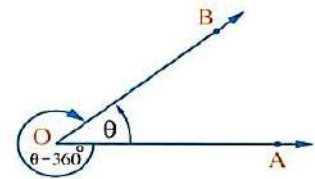


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So that :

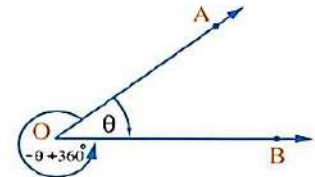
- 1 If the positive measure of the directed angle = θ , then the negative measure of the same directed angle = $\theta - 360^\circ$

For example : The negative measure of the directed angle of measure $210^\circ = 210^\circ - 360^\circ = -150^\circ$



- 2 If the negative measure of the directed angle = $-\theta$, then the positive measure of the same angle = $-\theta + 360^\circ$

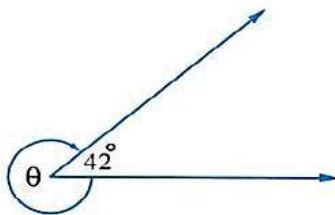
For example : The positive measure of the directed angle of measure $(-120^\circ) = -120^\circ + 360^\circ = 240^\circ$



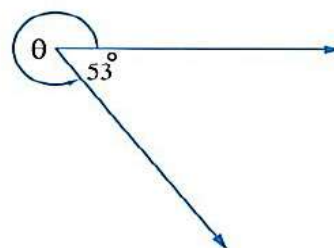
Example 1

Find the measure of the directed angle θ in each of the following figures :

1



2



Solution

- 1 $\therefore$ The rotation direction is clockwise
 $\therefore$ The measure of the angle is negative
 $\therefore \theta = 42^\circ - 360^\circ = -318^\circ$
- 2 $\therefore$ The rotation direction is anticlockwise
 $\therefore$ The measure of the angle is positive
 $\therefore \theta = -53^\circ + 360^\circ = 307^\circ$

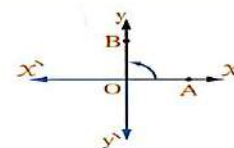
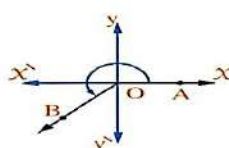
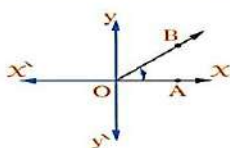
The standard position of the directed angle

A directed angle is in the standard position if the following two conditions are satisfied :

- 1 Its initial side lies on the positive direction of the X-axis.
- 2 Its vertex is the origin point of an orthogonal coordinate plane.

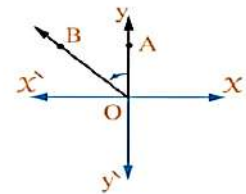
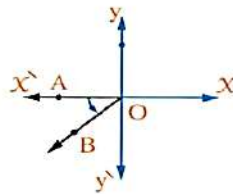
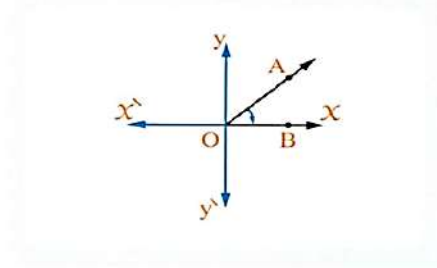
So that :

- All the following directed angles are **in the standard position** because they verify the two conditions :

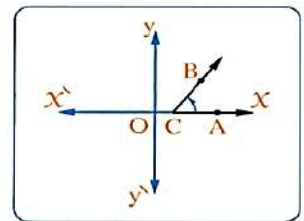


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- All the following directed angles are **not in the standard position** because the initial side does not lie on $\overrightarrow{OX}$



- The directed angle in the opposite figure is **not in the standard position** because its vertex is not the origin point O



Equivalent angles

- If we notice the directed angles in the standard position in the following figures :

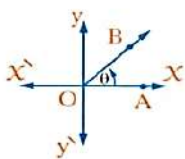


Fig. (1)

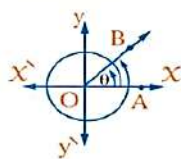


Fig. (2)

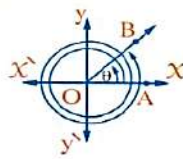


Fig. (3)

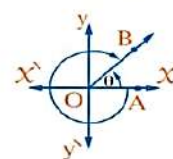


Fig. (4)

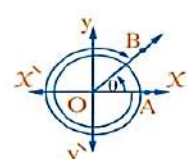


Fig. (5)

We notice the following :

- The angles in the five figures have the same terminal side $\overrightarrow{OB}$
- The measure of the angle in fig. (1) = θ ,
The measure of the angle in fig. (2) = $\theta + 360^\circ$,
The measure of the angle in fig. (3) = $\theta + 2 \times 360^\circ$,
The measure of the angle in fig. (4) = $-(360^\circ - \theta) = \theta - 360^\circ$,
The measure of the angle in fig. (5) = $-(2 \times 360^\circ - \theta) = \theta - 2 \times 360^\circ$

From this , we can conclude :

If θ is the measure of a directed angle in the standard position, then the angles whose measures are :

$(\theta \pm 360^\circ)$, $(\theta \pm 2 \times 360^\circ)$, $(\theta \pm 3 \times 360^\circ)$... , $(\theta \pm n \times 360^\circ)$, such that n is an positive integer have common terminal side.

These angles that have common terminal side are called "equivalent angles".

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Definition of the equivalent angles

Several directed angles in the standard position are said to be equivalent when they have one common terminal side.

Example 2

Determine two angles , one with positive measure and the other with negative measure having common terminal side for :

1 100°

2 -250°

Solution

1 An angle with positive measure = $100^\circ + 360^\circ = 460^\circ$

An angle with negative measure = $100^\circ - 360^\circ = -260^\circ$

2 An angle with positive measure = $-250^\circ + 360^\circ = 110^\circ$

An angle with negative measure = $-250^\circ - 360^\circ = -610^\circ$

Notice that

There are an infinite number of other positive and negative measures of angles having common terminal side.

Example 3

Determine the smallest positive measure for each of the angles whose measures are as follows :

1 -62°

2 -225°

3 530°

4 -790°

Solution

1 The smallest positive measure = $-62^\circ + 360^\circ = 298^\circ$

2 The smallest positive measure = $-225^\circ + 360^\circ = 135^\circ$

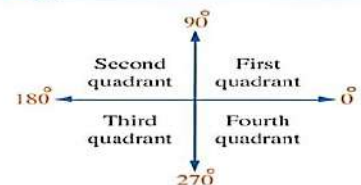
3 The smallest positive measure = $530^\circ - 360^\circ = 170^\circ$

4 The smallest positive measure = $-790^\circ + 3 \times 360^\circ = 290^\circ$

Angle position in the orthogonal coordinate plane

We know that the orthogonal coordinate plane is divided into four quadrants as in the opposite figure.

The position of the directed angle is determined by its terminal side when it is in its standard position.



If we draw the directed angle $\angle AOB$ in the standard position of positive measure θ , then :

The terminal side $\overline{OB}$ lies in a quadrant as follows :

First quadrant	Second quadrant	Third quadrant	Fourth quadrant
$\angle AOB$ lies in the first quadrant $0^\circ < \theta < 90^\circ$	$\angle AOB$ lies in the second quadrant $90^\circ < \theta < 180^\circ$	$\angle AOB$ lies in the third quadrant $180^\circ < \theta < 270^\circ$	$\angle AOB$ lies in the fourth quadrant $270^\circ < \theta < 360^\circ$

Remark

If the terminal side lies on one of the two axes , then the angle is called "quadrantal angle".

i.e. The angles whose measures are 0° , 90° , 180° , 270° , 360° are quadrantal angles.

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Example 4

Determine the quadrant in which each of the directed angles whose measures are as follows lies :

1 213°

2 132°

3 -310°

4 -12°

5 270°

6 964°

7 -1070°

Solution

1 $\because 180^\circ < 213^\circ < 270^\circ \quad \therefore$ The angle lies in the third quadrant.

2 $\because 90^\circ < 132^\circ < 180^\circ \quad \therefore$ The angle lies in the second quadrant.

3 The smallest positive measure $= -310^\circ + 360^\circ = 50^\circ$

$\because 0^\circ < 50^\circ < 90^\circ$

$\therefore$ The angle of measure 50° lies in the first quadrant

$\therefore$ The angle of measure -310° also lies in the first quadrant.

Notice that

To determine the quadrant which the directed angle lies in, we have to find the smallest positive measure of it.

4 The smallest positive measure $= -12^\circ + 360^\circ = 348^\circ$

$\because 270^\circ < 348^\circ < 360^\circ \quad \therefore$ The angle of measure 348° lies in the fourth quadrant.

$\therefore$ The angle of measure -12° also lies in the fourth quadrant.

5 270° is a quadrantal angle.

6 The smallest positive measure $= 964^\circ - 2 \times 360^\circ = 244^\circ$

$\because 180^\circ < 244^\circ < 270^\circ \quad \therefore$ The angle of measure 244° lies in the third quadrant.

$\therefore$ The angle of measure 964° also lies in the third quadrant.

7 The smallest positive measure $= -1070^\circ + 3 \times 360^\circ = 10^\circ$

$\because 0^\circ < 10^\circ < 90^\circ \quad \therefore$ The angle of measure 10° lies in the first quadrant.

$\therefore$ The angle of measure -1070° also lies in the first quadrant.

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Ex.1) Choose the correct answer :

1

The ordered pair $(\overrightarrow{OB}, \overrightarrow{OC})$ represents the directed angle

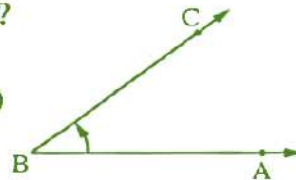
- (a) $\angle OBC$ (b) $\angle BOC$ (c) $\angle BCO$ (d) $\angle OCB$

2

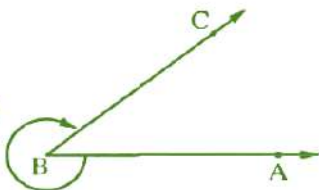
Which of the angles is not the directed $\angle ABC$?

- (a) $(\overrightarrow{BA}, \overrightarrow{BC})$

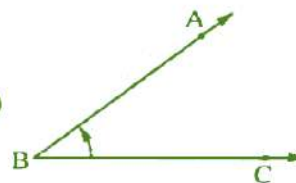
(b)



(c)



(d)



3

If θ is the smallest positive measure of a directed angle , then its negative measure is

- (a) $-\theta$ (b) $\theta - 180^\circ$ (c) $\theta - 360^\circ$ (d) $360^\circ - \theta$

4

If θ_1 is the positive measure of a directed angle and θ_2 is the negative measure of the same directed angle , then $\theta_1 - \theta_2 = \dots\dots\dots^\circ$

- (a) zero (b) ± 360 (c) 360 (d) -360

5

If θ is the directed angle , then the sum of its positive and negative measure

- (where θ is not zero angle)
(a) equal 360° (b) more than 360° (c) $\in]-360^\circ, 360^\circ[$ (d) $\in]0, 360^\circ[$

6

In the opposite figure :

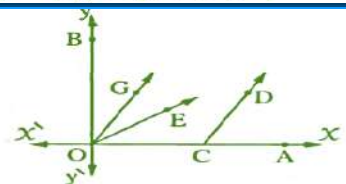
Which one of the following ordered pairs expresses a directed angle in its standard position ?

- (a) $(\overrightarrow{CA}, \overrightarrow{CD})$

- (b) $(\overrightarrow{OE}, \overrightarrow{OA})$

- (c) $(\overrightarrow{OB}, \overrightarrow{OG})$

- (d) $(\overrightarrow{OA}, \overrightarrow{OB})$



7

If the directed angle is in standard position , which of the following is correct ?

- ① its vertex is the origin.
② its initial side coincides the positive x -axis.
③ its measure is positive.

- (a) ① only (b) ① , ② only (c) ① , ③ only (d) All the previous.

8

It is said that the directed angles in the standard positions are equivalent if they have the same

- (a) initial side. (b) terminal side.
(c) vertex. (d) rotation direction.

9

If θ is the directed angle measure in standard position , $n \in \mathbb{Z}$, then the angles whose measures $(\theta \pm n \times 360^\circ)$ are called

- (a) equivalent. (b) quadrantal. (c) supplementary. (d) adjacent.

10

If A and B are the measures of two equivalent angles , then $-A$ and $-B$ are

- (a) supplementary. (b) equivalent. (c) complementary. (d) of sum -360°

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11	The quadrantal angle measure is multiple of
	(a) 360° (b) 180° (c) 90° (d) 60°
12	 The angle whose measure is 60° in the standard position is equivalent to the angle of measure
	(a) 120° (b) 240° (c) 300° (d) 420°
13	 The angle of measure 585° is equivalent to the angle in the standard position of measure
	(a) 45° (b) 135° (c) 225° (d) 315°
14	The angle whose measure is 950° is equivalent to the angle in the standard position of measure
	(a) 130° (b) -130° (c) 235° (d) -230°
15	All the following angles are equivalent to 75° in the standard position except
	(a) -285° (b) -645° (c) 285° (d) 435°
16	The quadrant in which the angle of measure 1670° lies is the
	(a) first. (b) second. (c) third. (d) fourth.
17	The angle whose measure is (-135°) lies in the quadrant.
	(a) first (b) second (c) third (d) fourth
18	 The angle whose measure is (-850°) lies in the quadrant.
	(a) first (b) second (c) third (d) fourth

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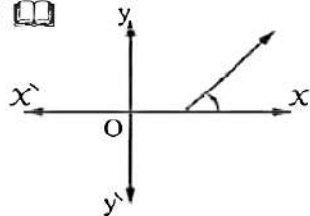
Ex.2) Answer the following:

1

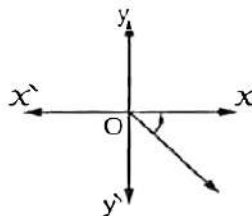
Which of the following directed angles is in its standard position ?

Explain your answer.

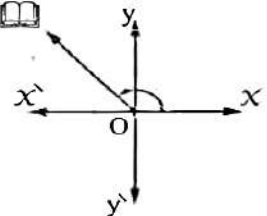
(1) 



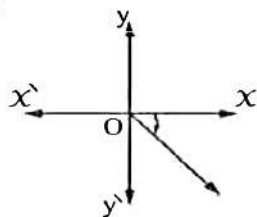
(2)



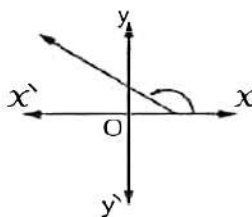
(3) 



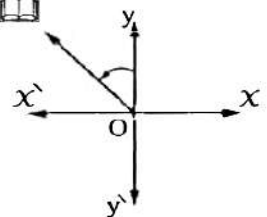
(4)



(5)



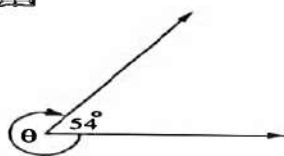
(6) 



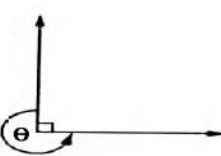
2

Find the measure of the directed angle θ in each of the following :

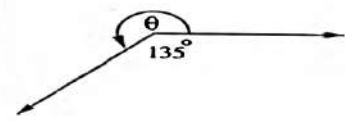
(1) 



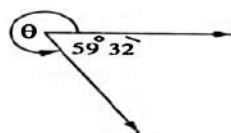
(2)



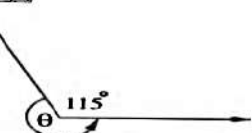
(3)



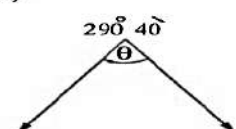
(4)



(5) 



(6)



3

 Show by drawing , each of the following angles in the standard position :

(1) 32°

(2) 140°

(3) -80°

(4) -110°

(5) -315°

4

Determine the quadrant in which each of the following angles lies :

(1)  24°

(2)  215°

(3) -50°

(4) -210°

5

Determine the smallest positive measure for each of the angles whose measures are as follows , then determine the quadrant in which each angle lies :

(1)  -56°

(2) 600°

(3)  -215°

(4) 940°

6

 Find two angles , one of them with positive measure and the other with negative measure having common terminal side for each of the following angles :

(1) 40°

(2) 150°

(3) -125°

(4) -240°

(5) -180°

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Lesson

2

Systems of measuring angle (Degree measure - Radian measure)

The unit of measurement of the degree measure

The degree is the unit of measuring the angle in the degree measure which is divided into 60 equal parts, each part is called a minute, and it is symbolized by $'$, also the minute is divided into 60 equal parts, each part is called a second and it is symbolized by $''$

i.e. $1^\circ = 60'$, $1' = 60''$

In this type of measuring angle, the protractor is used as an instrument for measuring angles in degrees.

Remember that

Calculator can be used to convert parts of degrees and minutes into minutes and seconds and vice versa

Such as

$$* 37 \frac{3}{8}^\circ = 37^\circ 22' 30''$$

$$* 70^\circ 37' 30'' = 70 \frac{5}{8}^\circ$$

$$37 \frac{3}{8} \text{ [0.] [.] [=]} 37^\circ 22' 30''$$

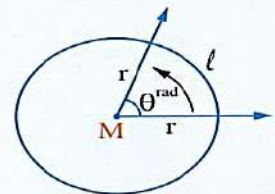
$$70 \text{ [0.] [.] [3] [7] [3] [0] [=] [SHIFT] [S \leftrightarrow D]} 70 \frac{5}{8}$$

Radian measure system

Definition

If θ^{rad} is the radian measure of a central angle in a circle of radius length r subtends an arc of length ℓ , then

$$\theta^{\text{rad}} = \frac{\ell}{r}$$



and since the radius length of the circle r is constant, then the radian measure of the central angle varies directly as the length of the subtended arc.

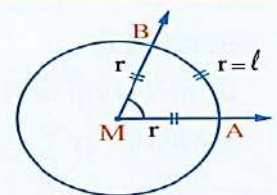
The unit of measurement of the radian measure

The radian angle is the unit of measuring the angle in the radian measure, and we can define the radian angle as follows which is denoted by (1^{rad}) and is read as one radian.

Definition

The radian angle is a central angle in a circle subtends an arc of length equals the length of the radius of the circle.

Notice : $\theta^{\text{rad}} = \frac{\ell}{r} \quad \therefore \theta^{\text{rad}} = \frac{r}{r} = 1^{\text{rad}}$



For example : The measure of the central angle that subtends an arc whose length equals double the length of the radius of its circle $= 2^{\text{rad}}$

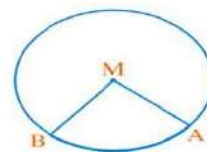
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The relation between the radian measure and the degree measure

You have known that , in a circle :

$$\frac{\text{Measure of the arc}}{\text{Measure of the circle}} = \frac{\text{Length of this arc}}{\text{Circumference of the circle}}$$

i.e. In the opposite figure : $\frac{m(\widehat{AB})}{360^\circ} = \frac{\text{Length of } \widehat{AB}}{2\pi r}$



$$\therefore m(\angle AMB) = m(\widehat{AB}) \quad \therefore \frac{m(\angle AMB)}{180^\circ} = \frac{\text{Length of } \widehat{AB}}{\pi r}$$

Assuming that : $m(\angle AMB)$ equals X° in degrees and equals θ^{rad} in radians and the length of $\widehat{AB} = l$

$$\therefore \frac{X^\circ}{180^\circ} = \frac{l}{\pi r} \quad , \quad \therefore \theta^{\text{rad}} = \frac{l}{r}$$

$$\therefore \frac{X^\circ}{180^\circ} = \frac{\theta^{\text{rad}}}{\pi} \quad \text{and from it} \quad \theta^{\text{rad}} = X^\circ \times \frac{\pi}{180^\circ} \quad , \quad X^\circ = \theta^{\text{rad}} \times \frac{180^\circ}{\pi}$$

Example 2

- Find the radian measure of the angle whose degree measure is $75^\circ 32' 15''$ approximating the result to the nearest thousandth.
- Find the degree measure of the angle whose radian measure is 2.38^{rad}

Solution

$$1 \quad \therefore \theta^{\text{rad}} = X^\circ \times \frac{\pi}{180^\circ} \quad \therefore \theta^{\text{rad}} = 75^\circ 32' 15'' \times \frac{\pi}{180^\circ} = 1.318^{\text{rad}}$$

$$2 \quad \therefore X^\circ = \theta^{\text{rad}} \times \frac{180^\circ}{\pi} \quad \therefore X^\circ = 2.38^{\text{rad}} \times \frac{180^\circ}{\pi} \approx 136^\circ 21' 50''$$

Enrichment information

There is another unit of measuring angles called (Grad) which equals $\frac{1}{200}$ of the measure of the straight angle.

If X, θ, y are the measures of three angles respectively in degrees , radian and grade

$$, \text{ then } \frac{X^\circ}{180^\circ} = \frac{\theta^{\text{rad}}}{\pi} = \frac{y^{\text{grad}}}{200}$$

Remarks

- If the radian measure of an angle equals π (radian) , then its degree measure

$$= \pi \times \frac{180^\circ}{\pi} = 180^\circ$$

i.e. π in radians is equivalent to 180° in degrees.

For example : $\frac{3}{5} \pi$ is equivalent to $\frac{3}{5} \times 180^\circ = 108^\circ$

- If the degree measure of an angle is known , and it is required to convert it into radian measure in terms of π , then we use the relation : $\theta^{\text{rad}} = X^\circ \times \frac{\pi}{180^\circ}$ without substituting with π

For example : • 18° is equivalent to $18^\circ \times \frac{\pi}{180^\circ} = \frac{\pi}{10}$

• 135° is equivalent to $135^\circ \times \frac{\pi}{180^\circ} = \frac{3}{4} \pi$

Example 3

Determine the quadrant in which the directed angle of each of the angles whose measures are as follows lies :

1 2.02^{rad}

2 -7.3^{rad}

3 $\frac{5}{4} \pi$

Solution

To determine the quadrant in which the directed angle lies , we find its degree measure :

$$1 \quad \therefore X^\circ = \theta^{\text{rad}} \times \frac{180^\circ}{\pi} = 2.02 \times \frac{180^\circ}{\pi} \approx 115^\circ 44' 15''$$

$\therefore$ The angle whose measure is 2.02^{rad} is equivalent to $115^\circ 44' 15''$ in degrees.

$\therefore$ The angle of measure $115^\circ 44' 15''$ lies in the second quadrant

$\therefore$ The angle of measure 2.02^{rad} lies in the second quadrant.

$$2 \quad \therefore X^\circ = -7.3^{\text{rad}} \times \frac{180^\circ}{\pi} \approx -418^\circ 15' 33''$$

$\therefore$ The angle of measure $-418^\circ 15' 33''$ is equivalent to

$$-418^\circ 15' 33'' + 2 \times 360^\circ = 301^\circ 44' 27''$$

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∴ The angle of measure $301^{\circ} 44' 27''$ lies in the fourth quadrant

∴ The angle of measure -7.3^{rad} lies in the fourth quadrant.

3 ∴ $\frac{5\pi}{4}$ is equivalent to $\frac{5}{4} \times 180^{\circ} = 225^{\circ}$

∴ The angle whose measure is 225° lies in the third quadrant.

∴ The angle whose measure is $\frac{5\pi}{4}$ lies in the third quadrant.

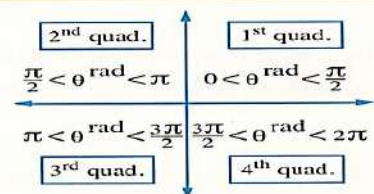
Remark

It is possible to determine the quadrant in which the directed angle - whose radian measure is known in terms of π - lies without converting to degrees using the opposite figure :

For example :

By using the opposite figure we can determine in which quadrant the angle whose measure is $\frac{5}{4}\pi$ in the last example lies where $\pi < \frac{5\pi}{4} < \frac{3\pi}{2}$

∴ The angle whose measure is $\frac{5}{4}\pi$ lies in the third quadrant.



Example 4

Find the length of the arc subtended by the central angle whose measure is $152^{\circ} 26' 17''$ drawn in a circle of radius length 10.5 cm. approximating the result to the nearest cm.

Solution

$$\therefore \theta^{\text{rad}} = x^{\circ} \times \frac{\pi}{180^{\circ}} = 152^{\circ} 26' 17'' \times \frac{\pi}{180^{\circ}} \approx 2.6605^{\text{rad}}$$

$$\therefore l = \theta^{\text{rad}} \times r = 2.6605 \times 10.5 \approx 28 \text{ cm.}$$

Example 5

Find each of the radian measure and the degree measure of the central angle subtending an arc of length 12.6 cm. in a circle of radius length 7.2 cm.

Solution

$$\theta^{\text{rad}} = \frac{l}{r} = \frac{12.6}{7.2} = 1.75^{\text{rad}}$$

$$\therefore x^{\circ} = 1.75^{\text{rad}} \times \frac{180^{\circ}}{\pi} \approx 100^{\circ} 16' 3''$$

Example 6

Find the circumference of the circle that has an inscribed angle of measure 30° subtending an arc of length 5 cm.

Solution

∴ The measure of the inscribed angle = 30°

∴ The measure of the corresponding central angle = 60°

$$\therefore \theta^{\text{rad}} = 60^{\circ} \times \frac{\pi}{180^{\circ}} = \frac{\pi}{3} \quad \therefore r = \frac{l}{\theta^{\text{rad}}} = 5 \div \left(\frac{\pi}{3}\right) = \frac{15}{\pi} \text{ cm.}$$

$$\therefore \text{The circumference of the circle} = 2\pi r = 2\pi \times \frac{15}{\pi} = 30 \text{ cm.}$$

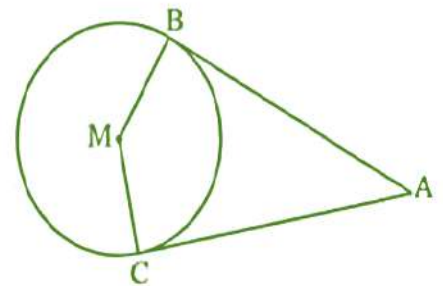
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Ex.1) Choose the correct answer :

1	The angle of measure $\frac{25\pi}{9}$ lies in the quadrant. (a) first (b) second (c) third (d) fourth
2	 The angle of measure $\frac{31\pi}{6}$ lies in the quadrant. (a) first (b) second (c) third (d) fourth
3	 The angle of measure $\frac{-9\pi}{4}$ lies in the quadrant. (a) first (b) second (c) third (d) fourth
4	If the degree measure of an angle is $43^\circ 12'$, then its radian measure is (a) 0.24^{rad} (b) 0.24π (c) 0.28^{rad} (d) 0.28π
5	The degree measure of the angle of measure $\frac{8\pi}{3}$ is (a) 540° (b) 820° (c) 150° (d) 480°
6	The sum of the measures of the angles of the quadrilateral in radian equals (a) 2π (b) π (c) $\frac{3\pi}{2}$ (d) 3π
7	 If the sum of measures of the interior angles of a regular polygon equals $180^\circ (n - 2)$ where n is the number of its sides, then the measure of the interior angle in radian of a regular pentagon equals (a) $\frac{\pi}{3}$ (b) $\frac{7\pi}{2}$ (c) $\frac{3\pi}{5}$ (d) $\frac{2\pi}{3}$
8	In a circle of diameter length 12 cm., the length of the arc subtended by a central angle of measure 60° equals cm. (a) 5π (b) 4π (c) 3π (d) 2π
9	The length of the arc subtended by a inscribed angle of measure 67.5° in a circle of radius length 8 cm. equal cm. (a) 3π (b) 6π (c) 1080 (d) 12π
10	 The measure of the central angle in a circle of radius length 15 cm. and opposite to an arc of length 5π cm. equals (a) 30° (b) 60° (c) 90° (d) 180°

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11	The measure of the central angle subtended by an arc of length equal the diameter length of the circle approximately to the nearest degree equal (a) 113° (b) 115° (c) 120° (d) 180°
12	If the measure of one of the angles of a triangle is 75° and the measure of another angle is $\frac{\pi}{3}$, then the radian measure of the third angle equals (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{4}$ (c) $\frac{\pi}{6}$ (d) $\frac{5\pi}{12}$
13	The string length of a simple pendulum is 14 cm. swings in an angle of measure $\frac{1}{10}\pi$, then its arc length $\simeq$ cm. (a) 4.6 (b) 4.4 (c) 4.2 (d) 4.8
14	ABCD is a cyclic quadrilateral, $m(\angle A) = 60^\circ$, then $m(\angle C) =$ (a) $\frac{\pi}{3}$ (b) $\frac{\pi}{6}$ (c) $\frac{2\pi}{3}$ (d) $\frac{5\pi}{6}$
15	The radian measure of a regular heptagon exterior angle equals (a) $\frac{1}{7}\pi$ (b) $\frac{2}{7}\pi$ (c) $\frac{3}{7}\pi$ (d) $\frac{4}{7}\pi$
16	In the opposite figure : If $\overline{AB}$, $\overline{AC}$ are two tangents to the circle M and $m(\angle A) = \frac{5}{12}\pi$ and the circle circumference = 96 cm. , then the smaller arc length $\widehat{BC} =$ (a) 20 (b) $\frac{28}{\pi}$ (c) 28 (d) 20π
17	The angle whose measure $30^\circ + 180^\circ(2n + 1)$ where $n \in \mathbb{Z}$, its radian measure is equivalent to (a) $\frac{\pi}{6}$ (b) π (c) $\frac{7}{6}\pi$ (d) $\frac{5}{3}\pi$



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Ex.2) Answer the following:

1 Find in terms of π the radian measure of each of the angles whose degree measures are as follows :

- | | | | |
|------------------|---------------------|-----------------|------------------|
| (1) 135° | (2) 90° | (3) 300° | (4) -235° |
| (5) -210° | (6) $112^\circ 30'$ | (7) 390° | (8) 780° |

2 Find the radian measure of each of the angles whose degree measures are as follows approximating the result to three decimal places :

- | | | |
|-------------------------|---------------------|--------------------------|
| (1) 58° | (2) 56.6° | (3) $37^\circ 15'$ |
| (4) $115^\circ 38' 6''$ | (5) $257^\circ 54'$ | (6) $160^\circ 50' 48''$ |

3 Find the degree measure (in degrees , minutes and seconds) of each of the angles whose radian measures are as follows :

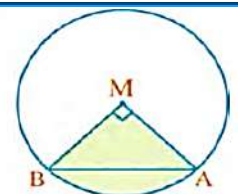
- | | | |
|--------------------------|-------------------------|----------------------------------|
| (1) $\frac{11\pi}{15}$ | (2) 0.72π | (3) 0.49^{rad} |
| (4) -1.67^{rad} | (5) 2.27^{rad} | (6) $-3\frac{1}{2}^{\text{rad}}$ |

4 Find the circumference of a circle which has an arc of length 12 cm. subtended by an inscribed angle of measure 45°

« 48 cm. »

5 In the opposite figure :

If the area of the right-angled triangle MAB at M equals 32 cm^2 , find the perimeter of the shaded area to the nearest hundredth.

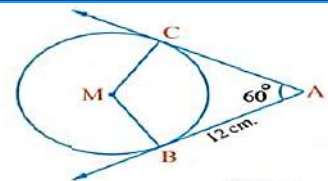


« 28.57 cm. »

6 In the opposite figure :

$\overrightarrow{AB}$, $\overrightarrow{AC}$ are two tangents to the circle M ,
 $m(\angle CAB) = 60^\circ$, $AB = 12 \text{ cm}$.

Find to the nearest integer the length of the greater arc $\widehat{BC}$

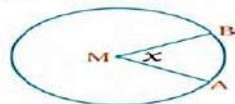


« 29 cm. »

Third Higher skills

1 Choose the correct answer from those given :

- (1) If an arc opposite to central angle of measure 72° was cut from a circle whose radius length 14 cm. and bent to form a circle , then the radius length of the resulted circle = cm.
 (a) 1.4 (b) 2.8 (c) 5.6 (d) 7
- (2) In the opposite figure :
 Circle whose centre M , the radius length 10 cm.
 , if the length of $\widehat{AB} \in]5, 6[$, then the value of x could be
 (a) 90° (b) 60° (c) 28° (d) 34°
- (3) If the ratio between measures of angles of a quadrilateral is 5 : 4 : 9 : 6 , then the measure of the smallest angle =rad
 (a) $\frac{\pi}{12}$ (b) $\frac{\pi}{3}$ (c) $\frac{5\pi}{12}$ (d) $\frac{3\pi}{4}$



2 A straight line makes an angle of radian measure $\frac{\pi}{3}$ with the positive direction of the X-axis in the standard position in the unit circle. Find the equation of the straight line.

« $y = \sqrt{3}x$ »

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Lesson

3

Trigonometric functions

We have studied before the basic trigonometric ratios of an acute angle and we have known that :

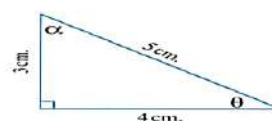
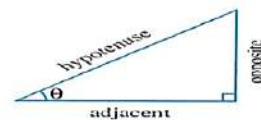
In any right-angled triangle :

$$\sin \theta = \frac{\text{Opposite}}{\text{Hypotenuse}}, \quad \cos \theta = \frac{\text{Adjacent}}{\text{Hypotenuse}}$$

$$, \tan \theta = \frac{\text{Opposite}}{\text{Adjacent}}$$

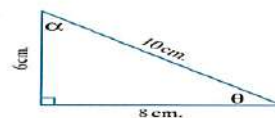
In the opposite figure :

$\sin \theta = \frac{3}{5}$	$\cos \theta = \frac{4}{5}$	$\tan \theta = \frac{3}{4}$
$\sin \alpha = \frac{4}{5}$	$\cos \alpha = \frac{3}{5}$	$\tan \alpha = \frac{4}{3}$



and if we draw another triangle similar to the previous triangle , we find that :

$\sin \theta = \frac{6}{10} = \frac{3}{5}$	$\cos \theta = \frac{8}{10} = \frac{4}{5}$	$\tan \theta = \frac{6}{8} = \frac{3}{4}$
$\sin \alpha = \frac{8}{10} = \frac{4}{5}$	$\cos \alpha = \frac{6}{10} = \frac{3}{5}$	$\tan \alpha = \frac{8}{6} = \frac{4}{3}$



From the previous , we deduce that :

1 $\sin \theta$, $\cos \theta$, $\tan \theta$ in the two triangles are equal.

i.e. The trigonometric ratio of the angle is constant and does not depend on the area of the triangle.

2 $\sin \theta \neq \sin \alpha$, $\cos \theta \neq \cos \alpha$, $\tan \theta \neq \tan \alpha$ in any of the two triangles.

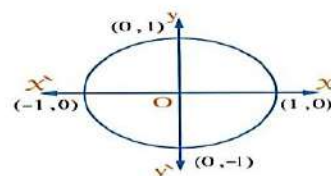
i.e. The trigonometric ratio is changed by the change of the angle which is known by "The trigonometric functions"

The unit circle

In the orthogonal coordinate system

the circle of centre at the origin point and

radius eq. 1 is the unit of length is called a unit circle.



Notice from the previous figure :

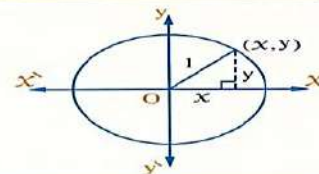
- The unit circle intersects the x-axis at two points which are (1 , 0) , (- 1 , 0)
- The unit circle intersects the y-axis at two points which are (0 , 1) , (0 , - 1)

Remark

If the point (X , y) ∈ the unit circle , then

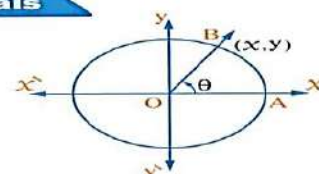
* $x^2 + y^2 = 1$ from Pythagoras' theorem.

* $x \in [- 1 , 1]$, $y \in [- 1 , 1]$



The basic trigonometric functions and their reciprocals

If we draw the directed angle AOB in the standard position and its terminal side intersects the unit circle at the point B (X , y) and if $m(\angle AOB) = \theta$, then we can define the following :



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First The basic trigonometric functions of the angle of measure θ are :

- 1 Cosine of the angle = x - coordinate of the point B **i.e.** $\cos \theta = x$
- 2 Sine of the angle = y - coordinate of the point B **i.e.** $\sin \theta = y$
- 3 Tangent of the angle = $\frac{y - \text{coordinate of the point B}}{x - \text{coordinate of the point B}}$ **i.e.** $\tan \theta = \frac{y}{x} = \frac{\sin \theta}{\cos \theta}$, where $x \neq 0$

Notice that The point B (x, y) can be written as $(\cos \theta, \sin \theta)$

Second The reciprocals of the basic trigonometric functions of the angle of measure θ are :

- 1 The secant of the angle ($\sec$) = $\frac{1}{x - \text{coordinate of the point B}}$
i.e. $\sec \theta = \frac{1}{x} = \frac{1}{\cos \theta}$, where $x \neq 0$
- 2 The cosecant of the angle ($\csc$) = $\frac{1}{y - \text{coordinate of the point B}}$
i.e. $\csc \theta = \frac{1}{y} = \frac{1}{\sin \theta}$, where $y \neq 0$
- 3 The cotangent of the angle ($\cot$) = $\frac{x - \text{coordinate of the point B}}{y - \text{coordinate of the point B}}$
i.e. $\cot \theta = \frac{x}{y} = \frac{\cos \theta}{\sin \theta} = \frac{1}{\tan \theta}$, where $y \neq 0$

Example 1

Find all trigonometric functions for an angle of measure θ which is drawn in the standard position and its terminal side intersects the unit circle at the point A in each of the following :

- 1 $A\left(\frac{3}{5}, \frac{4}{5}\right)$
- 2 $A(-1, 0)$
- 3 $A\left(-\frac{1}{2}, y\right)$, where $y > 0$
- 4 $A(-x, x)$ where $x > 0$

Solution

- 1 $\cos \theta = \frac{3}{5}$, $\sin \theta = \frac{4}{5}$, $\tan \theta = \frac{4}{5} \div \frac{3}{5} = \frac{4}{3}$
 $\sec \theta = \frac{5}{3}$, $\csc \theta = \frac{5}{4}$, $\cot \theta = \frac{3}{4}$
- 2 $\cos \theta = -1$, $\sin \theta = 0$, $\tan \theta = \frac{0}{-1} = 0$
 $\sec \theta = -1$, $\csc \theta = \frac{1}{0}$ (undefined), $\cot \theta = \frac{-1}{0}$ (undefined)
- 3 $\therefore x^2 + y^2 = 1$ $\therefore \left(-\frac{1}{2}\right)^2 + y^2 = 1$
 $\therefore y^2 = 1 - \frac{1}{4} = \frac{3}{4}$ $\therefore y = \pm \frac{\sqrt{3}}{2}$
 $\therefore y > 0$ $\therefore y = \frac{\sqrt{3}}{2}$ $\therefore A\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$
 $\therefore \cos \theta = -\frac{1}{2}$, $\sin \theta = \frac{\sqrt{3}}{2}$, $\tan \theta = \frac{\sqrt{3}}{2} \div -\frac{1}{2} = -\sqrt{3}$
 $\sec \theta = -2$, $\csc \theta = \frac{2}{\sqrt{3}}$, $\cot \theta = \frac{-1}{\sqrt{3}}$
- 4 $\therefore x^2 + y^2 = 1$ $\therefore (-x)^2 + x^2 = 1$
 $\therefore 2x^2 = 1$ $\therefore x^2 = \frac{1}{2}$
 $\therefore x = \pm \frac{1}{\sqrt{2}}$ $\therefore x > 0$
 $\therefore x = \frac{1}{\sqrt{2}}$ $\therefore A\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
 $\therefore \cos \theta = \frac{1}{\sqrt{2}}$, $\sin \theta = \frac{1}{\sqrt{2}}$, $\tan \theta = \frac{1}{\sqrt{2}} \div \frac{1}{\sqrt{2}} = 1$
 $\sec \theta = \sqrt{2}$, $\csc \theta = \sqrt{2}$, $\cot \theta = 1$

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Remark

The equivalent angles have the same trigonometric functions :

i.e. For all values of $n \in \mathbb{Z}$ (set of integers) , then

- $\cos(\theta + 2n\pi) = \cos \theta = x$, $\sec(\theta + 2n\pi) = \sec \theta = \frac{1}{x}$, where $x \neq 0$
- $\sin(\theta + 2n\pi) = \sin \theta = y$, $\csc(\theta + 2n\pi) = \csc \theta = \frac{1}{y}$, where $y \neq 0$
- $\tan(\theta + 2n\pi) = \tan \theta = \frac{y}{x}$, where $x \neq 0$, $\cot(\theta + 2n\pi) = \cot \theta = \frac{x}{y}$, where $y \neq 0$

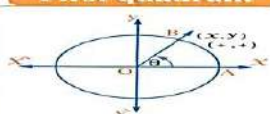
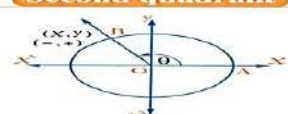
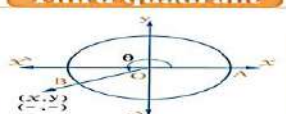
For example :

- $\cos 420^\circ = \cos(60^\circ + 360^\circ) = \cos 60^\circ$
- $\sec 840^\circ = \sec(120^\circ + 2 \times 360^\circ) = \sec 120^\circ$
- $\tan(-1500^\circ) = \tan(300^\circ - 5 \times 360^\circ) = \tan 300^\circ$

Signs of trigonometric functions

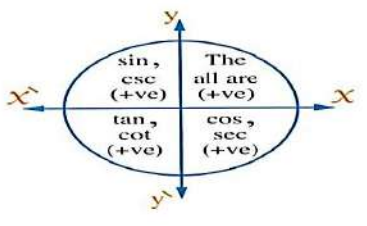
If $\angle AOB$ the directed is in its standard position and its terminal side intersects the unit circle at the point $B(x, y)$ and $m(\angle AOB) = \theta$, then

$\angle AOB$ lies in one of the quadrants as follows :

First quadrant	Second quadrant	Third quadrant	Fourth quadrant
			
$\theta \in]0, \frac{\pi}{2}[$ $x > 0, y > 0$ all the trigonometric functions are positive.	$\theta \in]\frac{\pi}{2}, \pi[$ $x < 0, y > 0$ $\sin \theta, \csc \theta$ are positive and the other functions are negative.	$\theta \in]\pi, \frac{3\pi}{2}[$ $x < 0, y < 0$ $\tan \theta, \cot \theta$ are positive and the other functions are negative.	$\theta \in]\frac{3\pi}{2}, 2\pi[$ $x > 0, y < 0$ $\cos \theta, \sec \theta$ are positive and the other functions are negative.

- We can summarize the previous results in the figure and in the following table :

Quadrant	The interval that θ belongs to	sign of $\cos, \sec$	sign of $\sin, \csc$	sign of $\tan, \cot$
First	$]0, \frac{\pi}{2}[$	+	+	+
Second	$]\frac{\pi}{2}, \pi[$	-	+	-
Third	$]\pi, \frac{3\pi}{2}[$	-	-	+
Fourth	$]\frac{3\pi}{2}, 2\pi[$	+	-	-



For example :

- $\tan 320^\circ$ is negative , because :
The angle of measure 320° lies in the fourth quadrant $270^\circ < 320^\circ < 360^\circ$
- $\sin 160^\circ$ is positive , because :
The angle of measure 160° lies in the second quadrant $90^\circ < 160^\circ < 180^\circ$

Remark

The trigonometric functions of the equivalent angles have the same sign.

Example 2

Determine the sign of each of the following trigonometric ratios :

- | | |
|----------------------|---------------------------|
| 1 $\sin 970^\circ$ | 2 $\cos \frac{7\pi}{3}$ |
| 3 $\tan(-200^\circ)$ | 4 $\csc(-\frac{8}{5}\pi)$ |

Solution

- 1 $\sin 970^\circ = \sin(250^\circ + 2 \times 360^\circ) = \sin 250^\circ$

$\therefore 180^\circ < 250^\circ < 270^\circ$

i.e. This angle lies in the third quadrant.

$\therefore \sin 250^\circ$ is negative.

$\therefore \sin 970^\circ$ is negative.

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2 $\cos \frac{7}{3} \pi = \cos \left(\frac{7}{3} \times 180^\circ \right) = \cos 420^\circ = \cos (60^\circ + 360^\circ) = \cos 60^\circ$
 $\therefore 0^\circ < 60^\circ < 90^\circ$

i.e. This angle lies in the first quadrant.

$\therefore \cos 60^\circ$ is positive.

$\therefore \cos \frac{7}{3} \pi$ is positive.

3 $\tan (-200^\circ) = \tan (-200^\circ + 360^\circ) = \tan 160^\circ$

$\therefore 90^\circ < 160^\circ < 180^\circ$

i.e. This angle lies in the second quadrant.

$\therefore \tan 160^\circ$ is negative.

$\therefore \tan (-200^\circ)$ is negative.

4 $\csc \left(-\frac{8}{5} \pi \right) = \csc \left(-\frac{8}{5} \times 180^\circ \right) = \csc (-288^\circ) = \csc (-288^\circ + 360^\circ) = \csc 72^\circ$

$\therefore 0^\circ < 72^\circ < 90^\circ$

i.e. This angle lies in the first quadrant.

$\therefore \csc 72^\circ$ is positive.

$\therefore \csc \left(-\frac{8}{5} \pi \right)$ is positive.

Example 3

If B $\left(x, \frac{1}{2} \right)$ is the point of intersection of the terminal side of the directed angle of measure θ in its standard position with the unit circle where $90^\circ < \theta < 180^\circ$, find the value of each of : $\cos \theta$ and $\tan \theta$

Solution

$\therefore 90^\circ < \theta < 180^\circ$

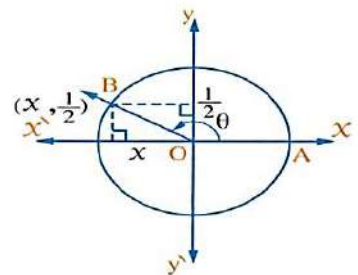
$\therefore$ B lies in the second quadrant

$\therefore$ for any point (X, y) on the unit circle, we get $x^2 + y^2 = 1$

$\therefore x^2 + \left(\frac{1}{2} \right)^2 = 1$

$\therefore x^2 = 1 - \frac{1}{4} = \frac{3}{4}$

$\therefore x = \pm \frac{\sqrt{3}}{2}$



$\therefore$ the point B $\left(x, \frac{1}{2} \right)$ lies in the second quadrant. $\therefore x = -\frac{\sqrt{3}}{2}$

$\therefore B = \left(-\frac{\sqrt{3}}{2}, \frac{1}{2} \right)$

$\therefore \cos \theta = -\frac{\sqrt{3}}{2}, \tan \theta = \frac{y}{x} = -\frac{1}{\sqrt{3}}$

Example 4

If $\theta \in \left] \frac{3}{2} \pi, 2\pi \right[$, $\cos \theta = \frac{5}{13}$, then find all trigonometric functions of θ

Solution

Let $m(\angle AOB) = \theta$ where θ is in the 4th quadrant and the point B is (x, y)

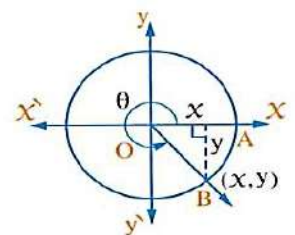
$\therefore x = \cos \theta = \frac{5}{13}, y = \sin \theta$ where $\sin \theta < 0$

$\therefore x^2 + y^2 = 1 \quad \therefore \left(\frac{5}{13} \right)^2 + \sin^2 \theta = 1$

$\therefore \sin^2 \theta = 1 - \frac{25}{169} = \frac{144}{169} \quad \therefore \sin \theta = -\frac{12}{13} \quad \therefore B = \left(\frac{5}{13}, -\frac{12}{13} \right)$

$\therefore$ then we get : $\tan \theta = \frac{y}{x} = -\frac{12}{5}$,

$\csc \theta = \frac{1}{y} = -\frac{13}{12}, \sec \theta = \frac{1}{x} = \frac{13}{5}$ and $\cot \theta = \frac{x}{y} = -\frac{5}{12}$

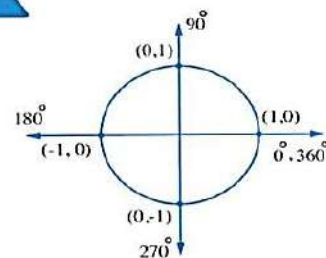


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The trigonometric ratios of some special angles

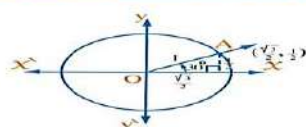
First The quadrantal angles (0° , 360° , 90° , 180° or 270°) :

The opposite figure illustrate the points of intersection of the terminal sides of the quadrantal angles with the unit circle, from which we can deduce the trigonometric ratios for these angles as shown in the following table :

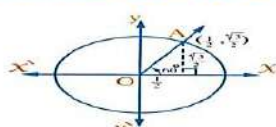


θ° in degree	θ in radian	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
0° or 360°	0 or 2π	0	1	0	undefined	1	undefined
90°	$\frac{\pi}{2}$	1	0	undefined	1	undefined	0
180°	π	0	-1	0	undefined	-1	undefined
270°	$\frac{3\pi}{2}$	-1	0	undefined	-1	undefined	0

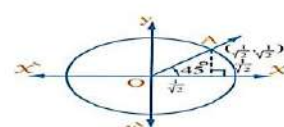
Second The angles of measures 30° , 60° and 45° :



θ in degree = 30°
 θ in radian = $\frac{\pi}{6}$



θ in degree = 60°
 θ in radian = $\frac{\pi}{3}$



θ in degree = 45°
 θ in radian = $\frac{\pi}{4}$

The previous figures show the points of intersection of the terminal side of each of the angles of measures 30° , 60° and 45° in the standard position with the unit circle, from which we can deduce the trigonometric ratios of these angles as shown in the following table :

θ° in degree	θ in radian	$\sin \theta$	$\cos \theta$	$\tan \theta$	$\csc \theta$	$\sec \theta$	$\cot \theta$
30°	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{3}}$	2	$\frac{2}{\sqrt{3}}$	$\sqrt{3}$
60°	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{2}{\sqrt{3}}$	2	$\frac{1}{\sqrt{3}}$
45°	$\frac{\pi}{4}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{\sqrt{2}}$	1	$\sqrt{2}$	$\sqrt{2}$	1

Example 5

Find the value of :

$$4 \sin 30^\circ \sin 90^\circ - \cos 0^\circ \sec 60^\circ + 5 \tan 45^\circ + 10 \cos^2 45^\circ \sin 270^\circ - \tan 30^\circ \sin 180^\circ$$

Solution

$$\begin{aligned} \text{The expression} &= 4 \times \frac{1}{2} \times 1 - 1 \times 2 + 5 \times 1 + 10 \times \left(\frac{1}{\sqrt{2}}\right)^2 \times (-1) - \frac{1}{\sqrt{3}} \times 0 \\ &= 2 - 2 + 5 - 5 - 0 = 0 \end{aligned}$$

Example 6

$$\text{Prove that : } \sin^2 60^\circ + \sin^2 45^\circ + \sin^2 30^\circ = \cos^2 \frac{\pi}{6} \sin \frac{\pi}{2} - \frac{1}{3} \tan^2 \frac{\pi}{3} \cos \pi + \cos^2 \frac{\pi}{3} \sin \frac{3\pi}{2}$$

Solution

$$\text{The left hand side} = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^2 + \left(\frac{1}{2}\right)^2 = \frac{3}{4} + \frac{1}{2} + \frac{1}{4} = \frac{3}{2}$$

$$\begin{aligned} \text{The right hand side} &= \cos^2 30^\circ \sin 90^\circ - \frac{1}{3} \tan^2 60^\circ \cos 180^\circ + \cos^2 60^\circ \sin 270^\circ \\ &= \left(\frac{\sqrt{3}}{2}\right)^2 \times 1 - \frac{1}{3} \times (\sqrt{3})^2 \times (-1) + \left(\frac{1}{2}\right)^2 \times (-1) = \frac{3}{4} + 1 - \frac{1}{4} = \frac{3}{2} \end{aligned}$$

$\therefore$ The two sides are equal.

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Ex.1) Choose the correct answer :

- (1) If θ is the measure of an angle in the standard position, its terminal side intersects the unit circle at the point $(\frac{\sqrt{3}}{2}, \frac{1}{2})$, then $\sin \theta = \dots\dots\dots$
 (a) $\frac{1}{2}$ (b) $\frac{\sqrt{3}}{2}$ (c) $\frac{1}{\sqrt{3}}$ (d) $\frac{2}{\sqrt{3}}$
- (2) If the terminal side of the angle whose measure θ drawn in the standard position intersect the unit circle at the point B $(\frac{-3}{5}, \frac{4}{5})$, then $\cot \theta = \dots\dots\dots$
 (a) $\frac{5}{4}$ (b) $\frac{-5}{3}$ (c) $\frac{-4}{3}$ (d) -0.75
- (3) If θ is a directed angle in the standard position its terminal side intersect the unit circle at $(\frac{-5}{13}, \frac{12}{13})$, then $\cos \theta - \sin \theta = \dots\dots\dots$
 (a) $\frac{17}{13}$ (b) $\frac{7}{13}$ (c) $\frac{-7}{13}$ (d) $\frac{-17}{13}$
- (4) A directed angle in the standard position its terminal side passes through the point $(3, 4)$, then its initial side intersect the unit circle at the point $\dots\dots\dots$
 (a) $(3, 0)$ (b) $(1, 0)$ (c) $(0.6, 0.8)$ (d) $(\frac{4}{3}, \frac{5}{3})$
- (5) If $\tan \theta = \frac{1}{2}$ where θ is an acute angle in standard position, then its terminal side intersects the unit circle at the point $\dots\dots\dots$
 (a) $(2, 1)$ (b) $(1, 2)$ (c) $(\frac{1}{\sqrt{5}}, \frac{2}{\sqrt{5}})$ (d) $(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}})$
- (6) If $\sin \theta = -1$, $\cos \theta = 0$, then the measure of angle $\theta = \dots\dots\dots$
 (a) $\frac{\pi}{2}$ (b) π (c) $\frac{3\pi}{2}$ (d) 2π
- (7) If $\csc \theta = 2$, where θ is a positive acute angle, then the measure of angle $\theta = \dots\dots\dots$
 (a) 15° (b) 30° (c) 45° (d) 60°
- (8) If $\cos \theta = \frac{1}{2}$, $\sin \theta = \frac{\sqrt{3}}{2}$, then the measure of angle $\theta = \dots\dots\dots$
 (a) $\frac{\pi}{3}$ (b) $\frac{5\pi}{6}$ (c) $\frac{5\pi}{3}$ (d) $\frac{11\pi}{6}$
- (9) If $\cos \theta = \frac{1}{2}$, $\sin \theta = \frac{-\sqrt{3}}{2}$, then $\tan \theta = \dots\dots\dots$
 (a) $\frac{\sqrt{3}}{2}$ (b) $\frac{-1}{2}$ (c) $\frac{-1}{\sqrt{3}}$ (d) $-\sqrt{3}$
- (10) If the terminal side of a directed angle in the standard position intersect the unit circle at the point $(\frac{1}{2}, \frac{\sqrt{3}}{2})$, then the measure of this angle = $\dots\dots\dots$
 (a) 150° (b) 30° (c) 60° (d) 210°
- (11) If $\cos \theta = \frac{\sqrt{3}}{2}$, where θ is a positive acute angle, then $\sin \theta = \dots\dots\dots$
 (a) $\frac{1}{2}$ (b) $\frac{1}{\sqrt{3}}$ (c) $\frac{2}{\sqrt{3}}$ (d) $\frac{\sqrt{3}}{2}$
- (12) If $\cos \theta > 0$, $\sin \theta < 0$, then θ lies in the $\dots\dots\dots$ quadrant.
 (a) first (b) second (c) third (d) fourth
- (13) If $\sin \theta = \frac{-1}{2}$, $\sec \theta = \frac{-2}{\sqrt{3}}$, then θ lies in the $\dots\dots\dots$ quadrant.
 (a) first (b) second (c) third (d) fourth
- (14) If θ is measure of an angle lies in the third quadrant, which of the following is always true?
 (a) $\sin \theta \cos \theta < 0$ (b) $\sec \theta \csc \theta < 0$ (c) $\tan \theta \cot \theta < 0$ (d) $\sin \theta \tan \theta < 0$
- (15) $2 \sin 45^\circ = \dots\dots\dots$
 (a) $\sin 90^\circ$ (b) $\frac{\sqrt{2}}{2}$ (c) $\sqrt{2}$ (d) 2
- (16) $\cot^2 30^\circ - \sec^2 60^\circ + \csc^2 45^\circ = \dots\dots\dots$
 (a) 1 (b) 0 (c) -1 (d) 2

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Ex.2) Answer the following:

- 1 Determine the signs of the following trigonometric ratios :

(1) $\cos 350^\circ$	(2) $\sec 265^\circ$	(3) $\sin \frac{5\pi}{4}$
(4) $\csc \frac{3\pi}{7}$	(5) $\tan 410^\circ$	(6) $\cos (-165^\circ)$
- 2 Find all trigonometric functions of the angle whose measure is θ drawn in the standard position , its terminal side intersects the unit circle at the point :

(1) $\left(\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$	(2) $\left(-\frac{3}{5}, -\frac{4}{5}\right)$	(3) $(0, -1)$
--	---	-----------------
- 3 If θ is the measure of a directed angle in the standard position and B is the intersection point of its terminal side with the unit circle , then find all trigonometric functions of the angle θ in each of the following cases :

(1) B (0.6 , y) , $y > 0$	(2) B (x , -0.6) , $x > 0$
(3) B $\left(-\frac{\sqrt{3}}{2}, y\right)$, where $90^\circ < \theta < 180^\circ$	(4) B $\left(x, \frac{\sqrt{5}}{3}\right)$, $x < 0$
(5) B (-1 , y)	(6) B ($-x, x$) , $x > 0$
- 4 Find the value of each of :

(1) $\tan 0^\circ + \tan 45^\circ + \tan 180^\circ$	(2) $\sin 180^\circ \cos 45^\circ - \cos 180^\circ \sin 45^\circ$
(3) $\sec \frac{\pi}{6} \tan \frac{\pi}{3} - \cot \frac{\pi}{3} \cos \frac{\pi}{6}$	(4) $\frac{4 \sin^2 30^\circ - 3 \tan 45^\circ \cos 0^\circ}{2 \cos 60^\circ + 2 \sin 45^\circ \cos 45^\circ}$
(5) $3 \sin 30^\circ \sin^2 60^\circ - \cos 0^\circ \sec 60^\circ + \sin 270^\circ \cos^2 45^\circ$	
- 5 Prove each of the following equalities :

(1) $2 \sin^2 90^\circ = -2 \cos 180^\circ$	(2) $3 \cos 30^\circ \tan 60^\circ - 2 \sec 45^\circ \csc 45^\circ = \frac{1}{2}$
(3) $3 \cot^2 45^\circ - 2 \sin 60^\circ \cos 30^\circ = \frac{3}{2} \sin^2 90^\circ$	(4) $\sec 30^\circ \tan 60^\circ + \csc^2 60^\circ - \tan^2 45^\circ = \frac{7}{3}$
(5) $\sin 60^\circ \cos 30^\circ - \cos 60^\circ \sin 30^\circ = \sin^2 \frac{\pi}{4}$	
- 6 Find the value of x if :

(1) $x \sin^2 \frac{\pi}{4} \cos \pi = \tan^2 \frac{\pi}{3} \sin \frac{3\pi}{2}$	« 6 »
(2) $x \sin \frac{\pi}{4} \cos \frac{\pi}{4} \cot \frac{\pi}{6} = \tan^2 \frac{\pi}{4} - \cos^2 \frac{\pi}{3}$	« $\frac{\sqrt{3}}{2}$ »

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Lesson

4

Related angles

Definition of the related angles

They are two angles the difference between their measures or the sum of their measures equals a whole number of right angles.

For example : The two angles of measures 30° , 210° are two related angles.
because : $210^\circ - 30^\circ = 180^\circ$ i.e. Two right angles.

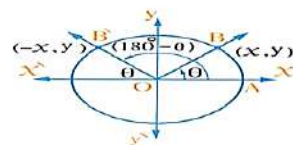
The relation between trigonometric functions of related angles

If the terminal side of the directed angle $\angle AOB$ in its standard position intersects the unit circle at the point $B(x, y)$ and $m(\angle AOB) = \theta$ such that $0^\circ < \theta < 90^\circ$, then :

1 Relation between trigonometric functions of related angles of measures θ , $(180^\circ - \theta)$:

If $\vec{B}(-x, y)$ is the image of the point $B(x, y)$ by reflection in the y-axis , then $m(\angle AOB)$ the directed = $(180^\circ - \theta)$ thus :

$$\begin{aligned} \sin(180^\circ - \theta) &= \sin \theta & \csc(180^\circ - \theta) &= \csc \theta \\ \cos(180^\circ - \theta) &= -\cos \theta & \sec(180^\circ - \theta) &= -\sec \theta \\ \tan(180^\circ - \theta) &= -\tan \theta & \cot(180^\circ - \theta) &= -\cot \theta \end{aligned}$$



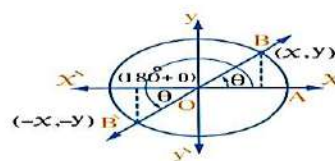
For example :

- $\sin 150^\circ = \sin(180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}$
- $\cos 120^\circ = \cos(180^\circ - 60^\circ) = -\cos 60^\circ = -\frac{1}{2}$
- $\cot 135^\circ = \cot(180^\circ - 45^\circ) = -\cot 45^\circ = -1$

2 Relation between trigonometric functions of related angles of measures θ , $(180^\circ + \theta)$:

If $\vec{B}(-x, -y)$ is the image of the point $B(x, y)$ by reflection in the origin point , then $m(\angle AOB)$ the directed = $(180^\circ + \theta)$ thus :

$$\begin{aligned} \sin(180^\circ + \theta) &= -\sin \theta & \csc(180^\circ + \theta) &= -\csc \theta \\ \cos(180^\circ + \theta) &= -\cos \theta & \sec(180^\circ + \theta) &= -\sec \theta \\ \tan(180^\circ + \theta) &= \tan \theta & \cot(180^\circ + \theta) &= \cot \theta \end{aligned}$$



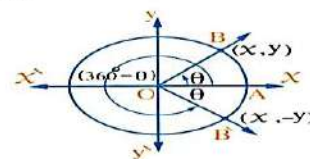
For example :

- $\sin 225^\circ = \sin(180^\circ + 45^\circ) = -\sin 45^\circ = -\frac{1}{\sqrt{2}}$
- $\sec 210^\circ = \sec(180^\circ + 30^\circ) = -\sec 30^\circ = -\frac{2}{\sqrt{3}}$
- $\tan 240^\circ = \tan(180^\circ + 60^\circ) = \tan 60^\circ = \sqrt{3}$

3 Relation between trigonometric functions of related angles of measures θ , $(360^\circ - \theta)$:

If $\vec{B}(x, -y)$ is the image of the point $B(x, y)$ by reflection in the x-axis , then $m(\angle AOB)$ the directed = $(360^\circ - \theta)$ thus :

$$\begin{aligned} \sin(360^\circ - \theta) &= -\sin \theta & \csc(360^\circ - \theta) &= -\csc \theta \\ \cos(360^\circ - \theta) &= \cos \theta & \sec(360^\circ - \theta) &= \sec \theta \\ \tan(360^\circ - \theta) &= -\tan \theta & \cot(360^\circ - \theta) &= -\cot \theta \end{aligned}$$



For example :

- $\sin 300^\circ = \sin(360^\circ - 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2}$
- $\tan 315^\circ = \tan(360^\circ - 45^\circ) = -\tan 45^\circ = -1$
- $\sec 330^\circ = \sec(360^\circ - 30^\circ) = \sec 30^\circ = \frac{2}{\sqrt{3}}$

Note

The angle of measure $(-\theta)$ is equivalent to the angle of measure $(360^\circ - \theta)$

From this , we can deduce :

The relation between trigonometric functions of related angles of measures θ , $(-\theta)$ as follows :

$$\begin{aligned} \sin(-\theta) &= -\sin \theta & \csc(-\theta) &= -\csc \theta \\ \cos(-\theta) &= \cos \theta & \sec(-\theta) &= \sec \theta \\ \tan(-\theta) &= -\tan \theta & \cot(-\theta) &= -\cot \theta \end{aligned}$$

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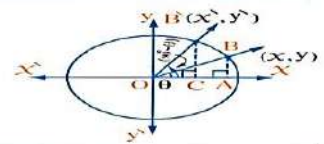
For example : • $\sin(-45^\circ) = -\sin 45^\circ = \frac{-1}{\sqrt{2}}$
• $\cot(-30^\circ) = -\cot 30^\circ = -\sqrt{3}$

• $\cos(-60^\circ) = \cos 60^\circ = \frac{1}{2}$

4 Relation between trigonometric functions of related angles of measures θ , $(90^\circ - \theta)$:

In the opposite figure :

The terminal side of the directed angle of measure $(90^\circ - \theta)$ in the standard position intersects the unit circle at the point $\tilde{B}(\tilde{x}, \tilde{y})$



From the figure geometry , we find that :

$$\triangle C\tilde{B}O \equiv \triangle AOB$$

$$\therefore C\tilde{B} = AO \quad , \quad \text{then } \tilde{y} = x$$

$$, CO = AB \quad , \quad \text{then } \tilde{x} = y$$

$$, \therefore \tan(90^\circ - \theta) = \frac{\tilde{y}}{\tilde{x}} = \frac{x}{y}$$

i.e. $\sin(90^\circ - \theta) = \cos \theta$

i.e. $\cos(90^\circ - \theta) = \sin \theta$

$$\therefore \tan(90^\circ - \theta) = \cot \theta$$

Similarly , it is possible to deduce the relations between the reciprocals of the trigonometric functions of the two angles of measures θ , $(90^\circ - \theta)$ as follows :

$$\sin(90^\circ - \theta) = \cos \theta \quad , \quad \csc(90^\circ - \theta) = \sec \theta$$

$$\cos(90^\circ - \theta) = \sin \theta \quad , \quad \sec(90^\circ - \theta) = \csc \theta$$

$$\tan(90^\circ - \theta) = \cot \theta \quad , \quad \cot(90^\circ - \theta) = \tan \theta$$

For example : • $\sin 70^\circ = \sin(90^\circ - 20^\circ) = \cos 20^\circ$

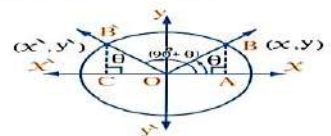
$$\bullet \frac{\sin 40^\circ}{\cos 50^\circ} = \frac{\sin(90^\circ - 50^\circ)}{\cos 50^\circ} = \frac{\cos 50^\circ}{\cos 50^\circ} = 1$$

$$\bullet \tan 10^\circ - \cot 80^\circ = \tan(90^\circ - 80^\circ) - \cot 80^\circ = \cot 80^\circ - \cot 80^\circ = 0$$

5 Relation between trigonometric functions of related angles of measures θ , $(90^\circ + \theta)$:

In the opposite figure :

The terminal side of the directed angle of measure $(90^\circ + \theta)$ in the standard position intersects the unit circle at the point $\tilde{B}(\tilde{x}, \tilde{y})$



From the figure geometry , we find that :

$$\triangle CO\tilde{B} \equiv \triangle ABO$$

$$\therefore C\tilde{B} = AO \quad , \quad \text{then } \tilde{y} = x$$

$$, OC = AB \quad , \quad \text{then } \tilde{x} = -y$$

$$, \therefore \tan(90^\circ + \theta) = \frac{\tilde{y}}{\tilde{x}} = \frac{x}{-y}$$

i.e. $\sin(90^\circ + \theta) = \cos \theta$

i.e. $\cos(90^\circ + \theta) = -\sin \theta$

$$\therefore \tan(90^\circ + \theta) = -\cot \theta$$

Similarly , it is possible to deduce the relations between the reciprocals of the trigonometric functions of the two angles of measures θ , $(90^\circ + \theta)$ as follows :

$$\sin(90^\circ + \theta) = \cos \theta \quad , \quad \csc(90^\circ + \theta) = \sec \theta$$

$$\cos(90^\circ + \theta) = -\sin \theta \quad , \quad \sec(90^\circ + \theta) = -\csc \theta$$

$$\tan(90^\circ + \theta) = -\cot \theta \quad , \quad \cot(90^\circ + \theta) = -\tan \theta$$

For example : • $\sin 120^\circ = \sin(90^\circ + 30^\circ) = \cos 30^\circ = \frac{\sqrt{3}}{2}$

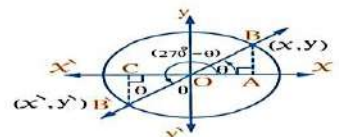
$$\bullet \cos 150^\circ = \cos(90^\circ + 60^\circ) = -\sin 60^\circ = \frac{-\sqrt{3}}{2}$$

$$\bullet \cot 135^\circ = \cot(90^\circ + 45^\circ) = -\tan 45^\circ = -1$$

6 Relation between trigonometric functions of related angles of measures θ , $(270^\circ - \theta)$:

In the opposite figure :

The terminal side of the directed angle of measure $(270^\circ - \theta)$ in the standard position intersects the unit circle at the point $\tilde{B}(\tilde{x}, \tilde{y})$



From the figure geometry , we find that :

$$\triangle CO\tilde{B} \equiv \triangle ABO$$

$$\therefore C\tilde{B} = AO \quad , \quad \text{then } \tilde{y} = -x$$

$$, CO = AB \quad , \quad \text{then } \tilde{x} = -y$$

$$, \therefore \tan(270^\circ - \theta) = \frac{\tilde{y}}{\tilde{x}} = \frac{-x}{-y} = \frac{x}{y}$$

i.e. $\sin(270^\circ - \theta) = -\cos \theta$

i.e. $\cos(270^\circ - \theta) = -\sin \theta$

$$\therefore \tan(270^\circ - \theta) = \cot \theta$$

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Similarly, it is possible to deduce the relations between the reciprocals of the trigonometric functions of the two angles of measures θ , $(270^\circ - \theta)$ as follows :

$$\begin{array}{ll} \sin (270^\circ - \theta) = -\cos \theta & , \quad \csc (270^\circ - \theta) = -\sec \theta \\ \cos (270^\circ - \theta) = -\sin \theta & , \quad \sec (270^\circ - \theta) = -\csc \theta \\ \tan (270^\circ - \theta) = \cot \theta & , \quad \cot (270^\circ - \theta) = \tan \theta \end{array}$$

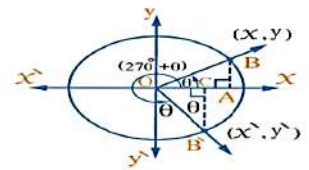
For example :

- $\sin 225^\circ = \sin (270^\circ - 45^\circ) = -\cos 45^\circ = -\frac{1}{\sqrt{2}}$
- $\tan 240^\circ = \tan (270^\circ - 30^\circ) = \cot 30^\circ = \sqrt{3}$
- $\csc 210^\circ = \csc (270^\circ - 60^\circ) = -\sec 60^\circ = -2$

7 Relation between trigonometric functions of related angles of measures θ , $(270^\circ + \theta)$:

In the opposite figure :

The terminal side of the directed angle of measure $(270^\circ + \theta)$ in the standard position intersects the unit circle at the point $B(\tilde{x}, \tilde{y})$



From the figure geometry, we find that :

$$\triangle COB \cong \triangle ABO$$

$$\therefore CB = AO \quad , \quad \text{then } \tilde{y} = -x$$

$$, CO = AB \quad , \quad \text{then } \tilde{x} = y$$

$$, \therefore \tan (270^\circ + \theta) = \frac{\tilde{y}}{\tilde{x}} = \frac{-x}{y}$$

$$\text{i.e. } \sin (270^\circ + \theta) = -\cos \theta$$

$$\text{i.e. } \cos (270^\circ + \theta) = \sin \theta$$

$$\therefore \tan (270^\circ + \theta) = -\cot \theta$$

Similarly, it is possible to deduce the relations between the reciprocals of the trigonometric functions of the two angles of measures θ , $(270^\circ + \theta)$ as follows :

$$\begin{array}{ll} \sin (270^\circ + \theta) = -\cos \theta & , \quad \csc (270^\circ + \theta) = -\sec \theta \\ \cos (270^\circ + \theta) = \sin \theta & , \quad \sec (270^\circ + \theta) = \csc \theta \\ \tan (270^\circ + \theta) = -\cot \theta & , \quad \cot (270^\circ + \theta) = -\tan \theta \end{array}$$

For example :

- $\sin 300^\circ = \sin (270^\circ + 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$
- $\sec 330^\circ = \sec (270^\circ + 60^\circ) = \csc 60^\circ = \frac{2}{\sqrt{3}}$
- $\cot 315^\circ = \cot (270^\circ + 45^\circ) = -\tan 45^\circ = -1$

We can summarize all the previous as follows (Where θ is the measure of an acute angle) :

For example :

$$\cos (180^\circ + \theta)$$

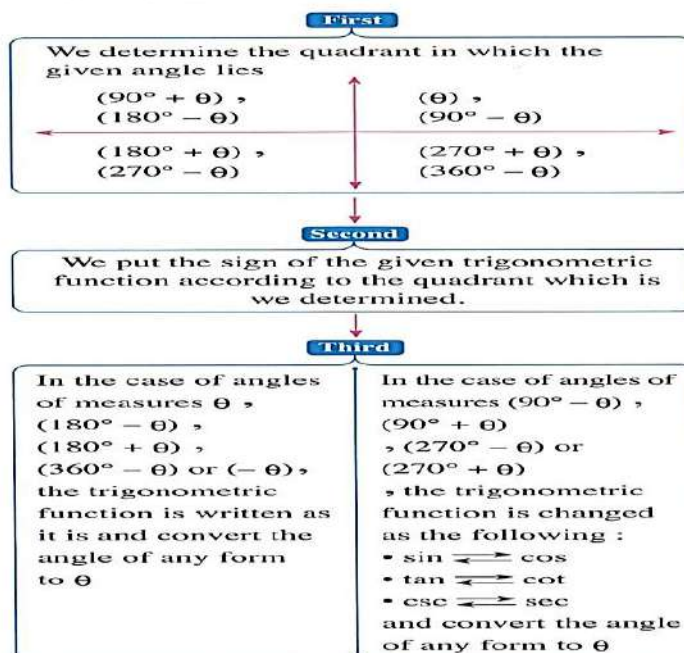
$(180^\circ + \theta)$ lies in the third quadrant

The function of cosine in the third quadrant is negative (-ve)

$$-\cos \theta$$

The function as it is because the measure of the angle is $(180^\circ + \theta)$

$$\therefore \cos (180^\circ + \theta) = -\cos \theta$$



For example :

$$\sin (90^\circ + \theta)$$

$(90^\circ + \theta)$ lies in the second quadrant

The function of sine in the second quadrant is positive (+ve)

$$+\cos \theta$$

The function is changed because the measure of the angle is $(90^\circ + \theta)$

$$\therefore \sin (90^\circ + \theta) = \cos \theta$$

SIMPLEST MATHS BOOKLETS

Finding a trigonometric function of an angle whose measure is given (α)

First If $0^\circ < \alpha < 360^\circ$ i.e. $\alpha \in]0, 2\pi[$

- 1 We determine the quadrant in which the angle lies , then determine the sign of the trigonometric function.
- 2 We convert the trigonometric function of α into the same trigonometric function of the angle θ and $\theta \in]0, \frac{\pi}{2}[$ as follows :
 - Put α in the form $(180^\circ - \theta)$ if α lies in the 2nd quadrant.
 - Put α in the form $(180^\circ + \theta)$ if α lies in the 3rd quadrant.
 - Put α in the form $(360^\circ - \theta)$ if α lies in the 4th quadrant.

Second If $\alpha > 360^\circ$ i.e. $\alpha > 2\pi$

- 1 Put α in the form of $(2n\pi + \theta)$ where $\theta \in]0, 2\pi[$, n is a positive integer , then the trigonometric function of the angle α is the same of the angle θ
- 2 Find the trigonometric function of the angle θ as in the first.

Third If α is (– ve) i.e. $\alpha < 0^\circ$

We follow one of the following two methods :

The first method

Apply the rule of the trigonometric function of the angle whose measure is negative , that is : $\sin(-\theta) = -\sin \theta$, $\cos(-\theta) = \cos \theta$, $\tan(-\theta) = -\tan \theta$ and so on , then we find the trigonometric function of the angle θ as in the first and the second.

The second method

Add to α an integer number of 2π (i.e. add to α the measures $360^\circ n$ or $2\pi n$ where $n \in \mathbb{Z}^+$) to get a positive angle $\theta \in]0, 2\pi[$, then we get the trigonometric function of the angle θ , the result is the same trigonometric function of the negative angle α

Notice that

There are other values for θ such as $\theta = 49^\circ$ or $\theta = 87^\circ$ that satisfy the equation and to find these values we have to generalize the previous remark to get a general solution for this kind of equations.

Generalizing the previous remark

- 1 If $\sin \alpha = \cos \beta$, then $\sin \alpha = \sin(90^\circ - \beta)$
 $\therefore \alpha = 90^\circ - \beta$ or $\alpha + 90^\circ - \beta = 180^\circ$
 $\therefore \alpha + \beta = 90^\circ$ | $\therefore \alpha - \beta = 90^\circ$

We can add the multiples of (360°) to the angle 90°

An Important Alert

On solving , we must start by sine angle α

- 2 In the same way , we can deduce the same rules if $\csc \alpha = \sec \beta$
- 3 If $\tan \alpha = \cot \beta$, then :

$$\begin{array}{lcl} \tan \alpha = \tan(90^\circ - \beta) & \text{or} & \tan \alpha = \tan(270^\circ - \beta) \\ \therefore \alpha = 90^\circ - \beta & & \therefore \alpha = 270^\circ - \beta \\ \therefore \alpha + \beta = 90^\circ & & \therefore \alpha + \beta = 270^\circ \end{array}$$

We can add the multiples of (360°) to the angles 90° and 270°

So , the general solution for any two angles α , β could be written as follows :

The general solution to solve the equations in the form :
 $\sin \alpha = \cos \beta$ or $\csc \alpha = \sec \beta$ or $\tan \alpha = \cot \beta$

- 1 If $\sin \alpha = \cos \beta$
 , then $\alpha \pm \beta = 90^\circ + 360^\circ n$ i.e. $\alpha \pm \beta = \frac{\pi}{2} + 2\pi n$ where $n \in \mathbb{Z}$
 i.e. The measure of angle of sine $\pm$ the measure of angle of cosine = $90^\circ + 360^\circ n$
- 2 If $\csc \alpha = \sec \beta$
 , then $\alpha \pm \beta = 90^\circ + 360^\circ n$ i.e. $\alpha \pm \beta = \frac{\pi}{2} + 2\pi n$ where $n \in \mathbb{Z}$
 , $\alpha \neq n\pi$, $\beta \neq (2n+1)\frac{\pi}{2}$
- 3 If $\tan \alpha = \cot \beta$
 , then $\alpha + \beta = 90^\circ + 180^\circ n$ i.e. $\alpha + \beta = \frac{\pi}{2} + \pi n$ where $n \in \mathbb{Z}$
 , $\alpha \neq (2n+1)\frac{\pi}{2}$, $\beta \neq n\pi$

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Ex.1) Choose the correct answer :

- (1) $\tan 42^\circ = \dots\dots\dots$
 (a) $\cot 42^\circ$ (b) $\tan 48^\circ$ (c) $\cot 48^\circ$ (d) $\csc 48^\circ$
- (2) $\frac{\sec 105^\circ}{\csc 15^\circ} = \dots\dots\dots$
 (a) $\frac{\sin 105^\circ}{\cos 15^\circ}$ (b) $\tan 135^\circ$ (c) $\cot 15^\circ$ (d) $\cos 90^\circ$
- (3) $\tan (180^\circ - \theta) = \dots\dots\dots$
 (a) $\tan \theta$ (b) $-\tan \theta$ (c) $\cot \theta$ (d) $-\cot \theta$
- (4) $\sec (90^\circ + \theta) = \dots\dots\dots$
 (a) $\csc (180^\circ - \theta)$ (b) $\csc (180^\circ + \theta)$ (c) $\csc (270^\circ - \theta)$ (d) $\csc (270^\circ + \theta)$
- (5) If $\sin \theta = \frac{3}{5}$, then $\cos (270^\circ - \theta) = \dots\dots\dots$
 (a) $\frac{3}{5}$ (b) $-\frac{3}{5}$ (c) $\frac{4}{5}$ (d) $-\frac{4}{5}$
- (6) $\cos (90^\circ - \theta) \times \csc \theta = \dots\dots\dots$
 (a) zero (b) 1 (c) -1 (d) $-\frac{4}{5}$
- (7) If $\frac{\sin 50^\circ}{\cos 40^\circ} + \frac{\sin 70^\circ}{\sin 110^\circ} = k$, then $k = \dots\dots\dots$
 (a) 1 (b) 2 (c) 3 (d) zero
- (8) $\tan (90^\circ - \theta) + \tan (90^\circ + \theta) = \dots\dots\dots$
 (a) $2 \cot \theta$ (b) $2 \tan \theta$ (c) zero (d) $\tan \theta + \cot \theta$
- (9) $\frac{\tan (45^\circ + X)}{\cot (45^\circ - X)} = \dots\dots\dots$
 (a) -1 (b) 1 (c) $\tan (90^\circ + X)$ (d) $\cot (90^\circ + X)$
- (10) $\sin (90^\circ - \theta) \sec (360^\circ - \theta) - \cos (270^\circ + \theta) \csc (180^\circ + \theta) = \dots\dots\dots$
 (a) -2 (b) -1 (c) 1 (d) 2
- (11) If $A + B = 90^\circ$, $\tan A = \frac{1}{3}$, then $\tan B = \dots\dots\dots$
 (a) $\frac{1}{3}$ (b) $\frac{2}{3}$ (c) 1 (d) 3
- (12) If $X + y = \frac{\pi}{2}$, then $\frac{\sin X - \sin y}{\cos X - \cos y} = \dots\dots\dots$
 (a) -1 (b) zero (c) 1 (d) 2
- (13) $\cos \theta + \cos (180^\circ - \theta) = \dots\dots\dots$
 (a) zero (b) 1 (c) $2 \cos \theta$ (d) $\cos \theta$
- (14) $\sin \theta + \cos (270^\circ + \theta) = \dots\dots\dots$
 (a) zero (b) 1 (c) $2 \sin \theta$ (d) $\sin \theta \cos \theta$
- (15) The simplest form of the expression :
 $\sin (180^\circ - \theta) + \cos (-60^\circ) + \cos (90^\circ + \theta) + \sin (-150^\circ) = \dots\dots\dots$
 (a) zero (b) 1 (c) -1 (d) $2 \sin \theta$
- (16) If $\cos \theta = -\sin 2\theta$, θ is the smallest positive measure, then $\theta = \dots\dots\dots^\circ$
 (a) 60 (b) 150 (c) 90 (d) 330
- (17) If $\sqrt{3} \csc \theta = -2$ where θ is the smallest positive angle, then $\theta = \dots\dots\dots$
 (a) 60° (b) 120° (c) 300° (d) 240°
- (18) If $\cos \theta = -\frac{1}{2}$, θ is measure of the smallest positive angle, then $\theta = \dots\dots\dots$
 (a) 60° (b) 120° (c) 240° (d) 300°
- (19) If $\cos (90^\circ + \theta) = \frac{\sqrt{3}}{2}$ where θ is the smallest positive angle, then $\theta = \dots\dots\dots$
 (a) 150° (b) 240° (c) 210° (d) 330°
- (20) If $\tan \theta = \tan (90 - \theta)$ where θ is an acute angle, then $\theta = \dots\dots\dots^\circ$
 (a) 15 (b) 30 (c) 45 (d) 60

SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

1

Find the value of each of the following :

(1) $\square \sin 150^\circ$

(2) $\square \sec 210^\circ$

(3) $\square \tan 240^\circ$

(4) $\square \cos (-150^\circ)$

(5) $\square \tan 225^\circ$

(6) $\square \csc \frac{11\pi}{6}$

(7) $\square \cot 780^\circ$

(8) $\square \cos (-900^\circ)$

(9) $\square \sin \left(-\frac{4\pi}{3} \right)$

(10) $\square \sec \left(-\frac{2\pi}{3} \right)$

(11) $\square \sec (-480^\circ)$

(12) $\square \sin \left(-\frac{7\pi}{4} \right)$

2

Find the value of each of the following :

(1) $\square \cos 120^\circ + \tan 225^\circ + \csc 330^\circ + \cos 420^\circ$

« -1 »

(2) $\square \sin 150^\circ \cos (-300^\circ) + \cos (930^\circ) \cot 240^\circ$

« $-\frac{1}{4}$ »

(3) $\square \tan \frac{2\pi}{3} \sec \frac{11\pi}{3} + \cot \frac{11\pi}{6} \csc \frac{19\pi}{6} + \tan \frac{25\pi}{6} \csc \left(-\frac{19\pi}{3} \right)$

« $-\frac{2}{3}$ »

3

Prove each of the following equalities :

(1) $\square \cos (-300^\circ) \sin 420^\circ - \cos 750^\circ \cos 660^\circ = \text{zero}$

(2) $\square \sin 600^\circ \cos (-30^\circ) + \sin 150^\circ \cos (-240^\circ) = -1$

(3) $\square \sin 150^\circ \tan 225^\circ + \cos 315^\circ \sec (-120^\circ) + \sin (-135^\circ) \csc 210^\circ = \frac{1}{2}$

4

If the terminal side of an angle of measure θ in its standard position intersects the unit circle at the point $\left(-\frac{3}{5}, \frac{4}{5} \right)$, find :

(1) $\square \sin (180^\circ + \theta)$

(2) $\square \cos \left(\frac{\pi}{2} - \theta \right)$

(3) $\square \tan (360^\circ - \theta)$

(4) $\square \csc \left(\frac{3\pi}{2} - \theta \right)$

(5) $\square \sec (\theta + \pi)$

(6) $\square \sin (\theta - \pi)$

5

If the directed angle of measure θ in the standard position, its terminal side passes by the point $\left(\frac{\sqrt{5}}{3}, \frac{2}{3} \right)$, find the following trigonometric functions :

(1) $\square \sin (270^\circ + \theta)$

(2) $\square \sec (270^\circ + \theta)$

(3) $\square \csc \left(\theta + \frac{\pi}{2} \right)$

6

Find one of the values of θ , where $0^\circ < \theta < 90^\circ$, which satisfies each of the following :

(1) $\square \sin (3\theta + 15^\circ) = \cos (2\theta - 5^\circ)$

« 16° »

(2) $\square \sec (\theta + 25^\circ) = \csc (\theta + 15^\circ)$

« 25° »

(3) $\square \tan (\theta + 20^\circ) = \cot (3\theta + 30^\circ)$

« 10° »

(4) $\square \cos \left(\frac{\theta + 20^\circ}{2} \right) = \sin \left(\frac{\theta + 40^\circ}{2} \right)$

« 60° »

(5) $\square \tan (\theta + 18^\circ 24') = \cot (\theta + 52^\circ 10')$

« $9^\circ 43'$ »

SIMPLEST MATHS BOOKLETS

7

 Find the general solution for each of the following equations :

(1) $\sin 2 \theta = \cos \theta$

(2) $\cos 5 \theta = \sin \theta$

8

Find the values of θ in the following cases where $\theta \in]0, \frac{\pi}{2}]$:

(1) $\csc (\theta + 15^\circ) = \sec 42^\circ$

(2) $\sin (\theta + 30^\circ) = \cos \theta$

(3)  $\sin \theta - \cos \theta = 0$

(4)  $\csc \left(\theta - \frac{\pi}{6} \right) = \sec \theta$

9

Find all values of θ , where $\theta \in]0, \frac{\pi}{2}]$ [which satisfies each of the following equations :

(1) $\tan \theta - 1 = 0$

(2) $2 \cos \theta - 1 = 0$

(3)  $2 \cos \left(\frac{\pi}{2} - \theta \right) = 1$

(4) $2 \sin \left(\frac{\pi}{2} - \theta \right) = \sqrt{3}$

10

 If $\cos \left(\frac{3\pi}{2} - \theta \right) = \frac{\sqrt{3}}{2}$, $\sin \left(\frac{\pi}{2} + \theta \right) = \frac{1}{2}$
 , find the measure of the smallest positive angle θ

« 300° »

SIMPLEST MATHS BOOKLETS

Lesson

5

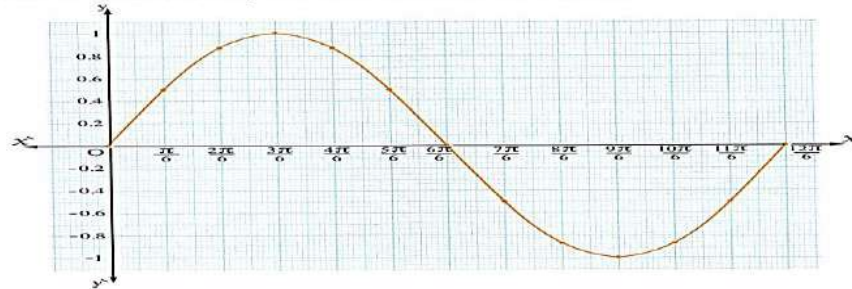
Graphing trigonometric functions

First Sine function : $f : f(\theta) = \sin \theta$

To represent the function $f : f(\theta) = \sin \theta$ graphically , we form the following table for some special values of θ , where $\theta \in [0 , 2\pi]$ and the corresponding values of $\sin \theta$

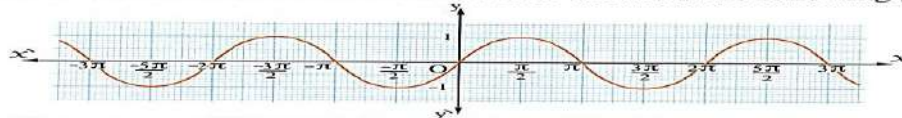
θ	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	2π
$\sin \theta$	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Represent all of the points that we get in the table on the coordinate axes and join them to get the curve of the function f on the interval $[0 , 2\pi]$



We notice that : The function is periodic and its period is 2π (i.e. 360°) where the curve of this function repeats itself on the intervals $[0 , 2\pi]$, $[2\pi , 4\pi]$, $[4\pi , 6\pi]$, ... and also on the intervals $[-2\pi , 0]$, $[-4\pi , -2\pi]$, $[-6\pi , -4\pi]$, ...

The general form of the curve of the sine function is as shown in the following graph :



From the previous , we can deduce the properties of the sine function $f : f(\theta) = \sin \theta$:

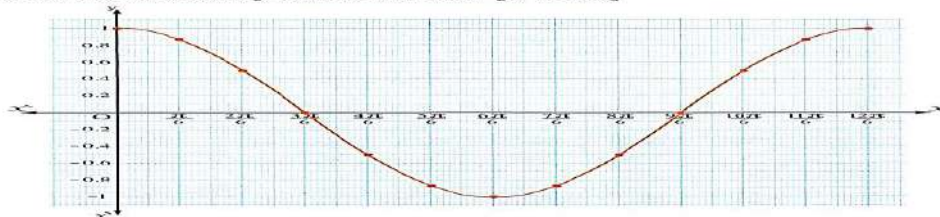
- 1 The domain of the sine function is $]-\infty , \infty[$
- 2 • The maximum value of the function is 1 and it happens when $\theta = \frac{\pi}{2} + 2n\pi$, $n \in \mathbb{Z}$
• The minimum value of the function is -1 and it happens when $\theta = \frac{3\pi}{2} + 2n\pi$, $n \in \mathbb{Z}$
- 3 The range of the function = $[-1 , 1]$
- 4 The function is periodic and its period is 2π (i.e. 360°)

Second Cosine function : $f : f(\theta) = \cos \theta$

To represent the function $f : f(\theta) = \cos \theta$ graphically , we form the following table for some special values of θ on the interval $[0 , 2\pi]$ and the corresponding values of $\cos \theta$

θ	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π	$\frac{7\pi}{6}$	$\frac{8\pi}{6}$	$\frac{9\pi}{6}$	$\frac{10\pi}{6}$	$\frac{11\pi}{6}$	2π
$\cos \theta$	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1

Represent all of the points that we get in the table on the coordinate axis and join them to get the curve of the function f on the interval $[0 , 2\pi]$



We notice that : The function is periodic and its period is 2π (i.e. 360°) where the curve of this function repeats itself on the intervals $[0 , 2\pi]$, $[2\pi , 4\pi]$, $[4\pi , 6\pi]$, ... and also on the intervals $[-2\pi , 0]$, $[-4\pi , -2\pi]$, $[-6\pi , -4\pi]$, ...

The general form of the curve of the cosine function is as shown in the following graph :



From the previous , we can deduce the properties of the cosine function $f : f(\theta) = \cos \theta$:

- 1 The domain of the cosine function is $]-\infty , \infty[$
- 2 • The maximum value of the function equals 1 and it happens when $\theta = 2n\pi$, where $n \in \mathbb{Z}$
• The minimum value of the function equals -1 and it happens when $\theta = \pi + 2n\pi$, where $n \in \mathbb{Z}$
- 3 The range of the function = $[-1 , 1]$
- 4 The function is periodic and its period is 2π (i.e. 360°)

Note

Each of the two functions $y = a \sin b\theta$, $y = a \cos b\theta$ is periodic , its period is $\frac{2\pi}{|b|}$ and its range is $[-a , a]$ where a is positive.

For example : The function $f : f(x) = 3 \sin 5x$ its range $[-3 , 3]$ and its period $\frac{2\pi}{5}$
If range of the function $f : f(x) = a \sin 5x$ is $[-3 , 3]$, then $a = \pm 3$

SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

- (1) The range of the function $f : f(\theta) = \sin \theta$ is
- (a) $\{-1, 1\}$ (b) $[-1, 1]$ (c) $]-1, 1[$ (d) $]-\infty, \infty[$
- (2) If $f(\theta) = \cos 5\theta$, then the range of the function is
- (a) $\{-5, 5\}$ (b) $[-1, 1]$ (c) $]-5, 5[$ (d) $[-5, 5]$
- (3) The range of the function $f : f(\theta) = 4 \sin 2\theta$ where $\theta \in [0, 2\pi]$ equal
- (a) $[-4, 4]$ (b) $]-4, 4[$ (c) $[-2, 2]$ (d) $]-2, 2[$
- (4) If $f(\theta) = \sin \theta$, $\theta \in [0, \pi[$, then the range of f is
- (a) $[-1, 1]$ (b) $[0, 1]$ (c) $[-1, 0]$ (d) $\mathbb{R}$
- (5) The range of the function $f : f(x) = \frac{\cos x}{5}$ where $x \in \mathbb{R}$ is
- (a) $[-\frac{1}{5}, \frac{1}{5}]$ (b) $[-1, 1]$ (c) $[-5, 5]$ (d) $[0, \frac{2}{5}]$
- (6) If the range of the function $f : f(\theta) = 2a \sin \theta$ is $[-6, 6]$, then $a = \dots\dots\dots$
- (a) 3 (b) -3 (c) 6 (d) a and b together.
- (7) The minimum value of the function $h : h(\theta) = 5 \cos 7\theta$ is
- (a) 5 (b) zero (c) -5 (d) -7
- (8) The minimum value of the function $f : f(\theta) = 1 + \sin 3\theta$ is
- (a) -3 (b) -2 (c) zero (d) -4

SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

- 1** Find the maximum and minimum values , then write the range of each of the following functions :

(1) $y = \frac{1}{2} \sin \theta$

(2) $y = \frac{1}{3} \sin 2 \theta$

(3) $y = 2 \sin 3 \theta$

- 2** Represent graphically each of the following functions and from the graph determine the minimum and maximum values of the function and write the range :

(1) $y = 4 \cos \theta$ where $\theta \in [0, 2\pi]$

(2) $y = 4 \sin \theta$ where $\theta \in [0, 2\pi]$

(3) $y = 2 \cos \theta$ where $\theta \in [-2\pi, 2\pi]$

(4) $y = 3 \sin \theta$ where $\theta \in [-2\pi, 2\pi]$

- 3** Represent graphically each of the following functions , and from the graph determine the minimum and maximum values of the function , and write the range :

(1) $y = \cos 3 \theta$

where $0^\circ \leq \theta \leq 120^\circ$

(2) $y = 5 \sin 2 \theta$

where $0^\circ \leq \theta \leq 180^\circ$

- 4**  Use the graph calculator or graphing program on your computer to graph each of the functions : $y = 4 \cos \theta$, $y = 3 \sin \theta$, then find from the graph :

(1) The range of the function.

(2) The maximum and minimum values of the function.

SIMPLEST MATHS BOOKLETS

Lesson

6

Finding the measure of an angle given the value of one of its trigonometric ratios

* We have studied that if $y = \sin \theta$, then it is possible to find the value of y if the value of θ is known

i.e. If $\theta = 30^\circ$, then $y = \sin 30^\circ = \frac{1}{2}$

* There is an inverse form is used to find the value of θ if the value of y is known, which is $\theta = \sin^{-1} y$

i.e. If $y = \frac{1}{2}$, then $\theta = \sin^{-1} \left(\frac{1}{2} \right) = 30^\circ$

Notice that

We use the calculator for the value of the trigonometric function is neither for a special angle nor a relative angle for a special angle.

Remark

The functions : $\theta = \sin^{-1} x$, $\theta = \cos^{-1} x$, $\theta = \tan^{-1} x$ are known as inverse functions of the basic trigonometric functions, these functions give a unique value of the variable θ for each value of the variable x and determine θ in a certain range according to the properties of each function so,

For example :

$$\sin^{-1} \left(-\frac{1}{2} \right) = -30^\circ$$

$$\cos^{-1} \left(\frac{1}{2} \right) = 60^\circ$$

i.e. (unique value $\in [-\frac{\pi}{2}, \frac{\pi}{2}]$)

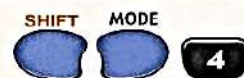
i.e. (unique value $\in [0, \pi]$)

Note

In the previous example :

$\theta = \sin^{-1} \frac{3}{4}$, we can get θ in radian directly using the calculator as follows :

1 Press , in succession, from left to right to convert the calculator from degree (Deg) system into radian (Rad) system.



2 Find θ in radian directly by pressing in succession from left to right



$$\therefore \theta^{\text{rad}} \approx 0.848$$

SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

(1) If $\theta = \sin^{-1} \frac{\sqrt{3}}{2}$, then $\theta = \dots\dots\dots$

- (a) 60° (b) 120° (c) 240° (d) 300°

(2) If $\csc \theta = -2$, $270^\circ < \theta < 360^\circ$, then $\theta = \dots\dots\dots$

- (a) 30° (b) 300° (c) 330° (d) 150°

(3) If $\tan \theta = \frac{-1}{\sqrt{3}}$, $90^\circ < \theta < 180^\circ$, then $\theta = \dots\dots\dots$

- (a) 30° (b) 120° (c) 150° (d) 210°

(4) If $\tan \theta = 1.8$ and $90^\circ \leq \theta \leq 360^\circ$, then $\theta \approx \dots\dots\dots$

- (a) $60^\circ 57'$ (b) $119^\circ 3'$ (c) $240^\circ 57'$ (d) $299^\circ 3'$

(5) If $y = \sin(90^\circ - \theta)$, then $\theta = \dots\dots\dots$

- (a) $\sin^{-1} y$ (b) $\cos^{-1} y$ (c) $\sin^{-1} \theta$ (d) $\cos^{-1} \theta$

(6) If $\csc \theta = -\sqrt{2}$, then each of the following could be a value of θ except $\dots\dots\dots$

- (a) 45° (b) -45° (c) -135° (d) 225°

(7) $\sin^{-1} 0.7 \approx \dots\dots\dots$

- (a) $44^\circ 25' 37''$ (b) $135^\circ 34' 23''$ (c) $224^\circ 25' 37''$ (d) $315^\circ 34' 23''$

(8) $\sin^{-1}(-0.6) \approx \dots\dots\dots$

- (a) -36.87° (b) 143.13° (c) 216.87° (d) 323.13°

(9) If $\cos \theta = 0.436$, where θ is the measure of the smallest positive angle, then $\theta \approx \dots\dots\dots$

- (a) $64^\circ 9'$ (b) $115^\circ 51'$ (c) $244^\circ 9'$ (d) $295^\circ 51'$

(10) If $\sin \theta = \frac{-1}{2}$ where θ is the measure of the smallest positive angle, then $\theta = \dots\dots\dots$

- (a) -30° (b) 30° (c) 210° (d) 150°

(11) If the terminal side of a directed angle θ in the standard position intersect the unit circle at $(-\frac{\sqrt{3}}{2}, y)$ where $y \in \mathbb{Z}^+$, then $\theta = \dots\dots\dots$

- (a) 30° (b) 150° (c) 210° (d) 330°

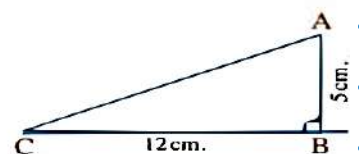
(12) In the opposite figure :

$m(\angle ACB) = \dots\dots\dots$

- (a) $\tan^{-1}(\frac{12}{5})$ (b) $\sin^{-1}(\frac{12}{13})$
(c) $\csc^{-1}(\frac{12}{13})$ (d) $\cos^{-1}(\frac{12}{13})$

(13) $\cos(\frac{1}{2})^\circ \times \cos^{-1}(\frac{1}{2}) \approx \dots\dots\dots$

- (a) 1 (b) $\frac{1}{4}$ (c) 60° (d) $\cos \frac{1}{4}$



SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

1 Find in degrees the measure of the smallest positive angle θ satisfying :

(1) $\sin \theta = 0.6$

(2) $\cos \theta = 0.7865$

(3) $\tan \theta = 2.4577$

(4) $\tan \theta = -0.8227$

(5) $\sin \theta = -0.4652$

(6) $\cos \theta = -0.5206$

(7) $\cot \theta = 3.6218$

(8) $\cot \theta = -1.4612$

(9) $\sec \theta = 1.0478$

(10) $\csc \theta = -2.5466$

(11) $\sec \theta = -3.57$

(12) $\csc \theta = 2.9811$

2 If $0^\circ < \theta < 360^\circ$, find θ which satisfies each of the following :

(1) $\sin \theta = 0.86603$

(2) $\cos \theta = -0.4752$

(3) $\csc \theta = -1.2576$

(4) $\tan \theta = 1.5417$

(5) $\cos \theta = -0.642$

(6) $\sec \theta = 2.0515$

(7) $\csc \theta = -1.8715$

(8) $\cot \theta = -2.7012$

(9) $\tan \theta = -2.1456$

3 If the terminal side of angle θ in the standard position intersects the unit circle at point B, then find $m(\angle \theta)$ where $0^\circ < \theta < 360^\circ$ when :

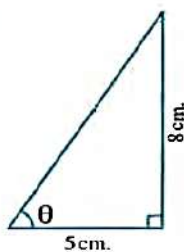
(1) $B\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

(2) $B\left(\frac{-1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$

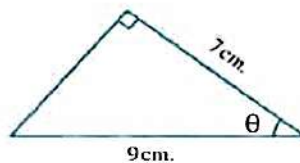
(3) $B\left(\frac{6}{10}, -\frac{8}{10}\right)$

4 Find the degree measure of the angle θ in each of the following figures :

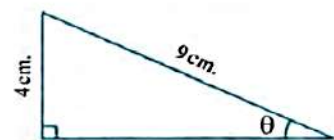
(1)



(2)



(3)



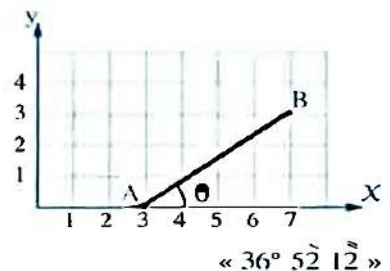
5 If $\sin \theta = \frac{1}{3}$ and $90^\circ \leq \theta \leq 180^\circ$:

(1) Calculate the measure of the angle θ to the nearest second.

(2) Find the value of each of the following : $\cos \theta$, $\tan \theta$, $\sec \theta$

11 The opposite figure represents a line segment joining between the two points A (3, 0), B (7, 3)

Find the measure of the angle θ included between $\overline{AB}$ and the X-axis.



Unit 3

SIMILARITY

1

Similarity of polygons.

2

Similarity of triangles

3

The relation between the areas of two similar polygons

4

Applications of similarity in the circle.



SIMPLEST MATHS BOOKLETS

Lesson

1

Similarity of polygons

Definition

Two polygons M_1 and M_2 (of same number of sides) are said to be similar if the following two conditions satisfied together :

- 1 Their corresponding angles are congruent.
- 2 The lengths of their corresponding sides are proportional.

In this case , we shall write :

The polygon $M_1 \sim$ the polygon M_2

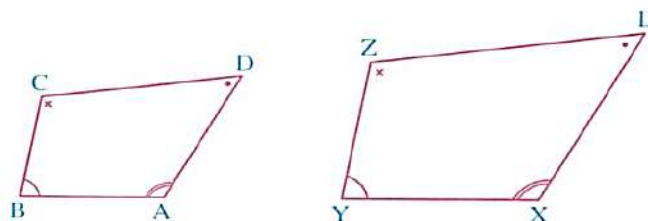
That means the polygon M_1 is similar to the polygon M_2

In the opposite figure , if :

- 1 $m(\angle A) = m(\angle X)$
 $, m(\angle B) = m(\angle Y)$
 $, m(\angle C) = m(\angle Z)$
 $, m(\angle D) = m(\angle L)$

$$2 \frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX}$$

Then the polygon $ABCD \sim$ the polygon $XYZL$



Remark 1

On writing the similar polygons , it is prefer to write them according to the order of their corresponding vertices to make it easy to deduce the equal angles in measure and write the proportion of corresponding side lengths.

Remark 2

If the polygon $ABCD \sim$ the polygon $XYZL$, then :

$$\frac{AB}{XY} = \frac{BC}{YZ} = \frac{CD}{ZL} = \frac{DA}{LX} = K \text{ (similarity ratio or scale factor of similarity) , } K > 0$$

If the scale factor of similarity of polygon $ABCD$ to polygon $XYZL = K$

$\therefore$ The scale factor of similarity of polygon $XYZL$ to polygon $ABCD = \frac{1}{K}$

Remark 3

Let K be the similarity ratio of polygon M_1 to polygon M_2 :

- If $K > 1$, then polygon M_1 is an enlargement of polygon M_2 , where K is called the enlargement ratio.
- If $0 < K < 1$, then polygon M_1 is a shrinking to polygon M_2 , where K is called the shrinking ratio.
- If $K = 1$, then polygon M_1 is congruent to polygon M_2

In general , you can use the similarity ratio in calculation of the dimensions of similar figures.

Remark 4

In order that two polygons are similar , the two conditions should be verified together and verifying one of them only is not enough to be similar.

SIMPLEST MATHS BOOKLETS

Remark 5

The congruent polygons are similar but it's not necessary that similar polygons are congruent.

Remark 6

If each of two polygons is similar to a third polygon, then they are similar.

i.e. If polygon $M_1 \sim$ polygon M_3 , polygon $M_2 \sim$ polygon M_3 , then polygon $M_1 \sim$ polygon M_2

Remark 7

All regular polygons of the same number of sides are similar.

For example : • All equilateral triangles are similar. • All squares are similar.

The ratio between the perimeters of two similar polygons = the ratio between the lengths of two corresponding sides of them.

SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

- (1) If K is the scale factor of similarity of polygon M_1 to polygon M_2 and $0 < K < 1$, then the polygon M_1 is to polygon M_2
 (a) congruent to (b) enlargement (c) minimization (d) of double area
- (2) If k is the scale factor of similarity of polygon M_1 to polygon M_2 and the polygon M_1 is minimization to polygon M_2 , then K may be equal
 (a) 1 (b) $\frac{3}{5}$ (c) $\frac{3}{2}$ (d) $\frac{4}{3}$
- (3) If K_1 is the scale factor of similarity of polygon M_1 to polygon M_2 and K_2 is the scale factor of similarity of polygon M_2 to polygon M_3 , then the scale factor of similarity of polygon M_1 to polygon M_3 is
 (a) $K_1 + K_2$ (b) $K_1 K_2$ (c) $\frac{K_1}{K_2}$ (d) $\frac{K_2}{K_1}$
- (4) The two similar polygons are congruent if the scale factor K satisfies
 (a) $K = \frac{1}{2}$ (b) $K = 1$ (c) $K > 1$ (d) $0 < K < 1$
- (5) If $\triangle ABC \sim \triangle DEF$, $BC = 3 EF$, then the scale factor of similarity of the two triangles =
 (a) $\frac{2}{3}$ (b) $\frac{1}{2}$ (c) 1 (d) 3
- (6) The scale factor of similarity between the square $ABCD$ and the square $XYZL$ equals each of the following except
 (a) $AC : XZ$ (b) $AB : YZ$ (c) $(AB)^2 : (XY)^2$ (d) $BC : YZ$
- (7) To make two polygons M_1 and M_2 similar, it is sufficient to have
 (a) their corresponding angles are equal in measures only.
 (b) their corresponding sides are in proportion only.
 (c) (a) and (b) together. (d) nothing of the previous.
- (8) To make two rhombuses $ABCD$, $XYZL$ similar it is sufficient to have
 (a) $m(\angle A) = 60^\circ$, $m(\angle Y) = 120^\circ$ only.
 (b) the perimeter of rhombus $ABCD = 2$ the perimeter of the rhombus $XYZL$ only.
 (c) (a) and (b) together. (d) nothing of the previous.
- (9) Which of the following statements is not true ?
 (a) each two squares are similar.
 (b) each two equilateral triangles are similar.
 (c) each two rhombuses are similar.
 (d) each two regular polygons with the same number of sides are similar.
- (10) The true statement from the following is
 (a) all the isosceles triangles are similar.
 (b) all the right angled triangles are similar.
 (c) all the squares uses are similar. (d) all the regular polygons are similar.
- (11) Which of the following statements is true ?
 (a) all the regular polygons are similar.
 (b) all the squares are congruent.
 (c) all the equilateral triangles are similar.
 (d) all the rhombuses are similar.
- (12) If M_1 , M_2 are two similar polygons and the lengths of two corresponding sides are 20 cm, 16 cm respectively, then the perimeter of polygon M_1 : the perimeter of M_2 =
 (a) 25 : 16 (b) 41 : 9 (c) 9 : 41 (d) 5 : 4
- (13) Two similar polygons, the ratio between their perimeters equal 4 : 9, then the ratio between the lengths of two corresponding sides is
 (a) 4 : 9 (b) 2 : 3 (c) 16 : 81 (d) 9 : 4

SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

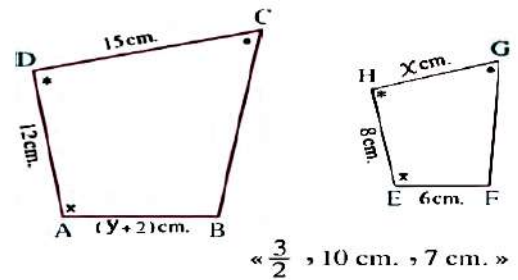
1

In the opposite figure :

Polygon ABCD ~ polygon EFGH

(1) Find : The scale factor of similarity of polygon ABCD to polygon EFGH

(2) Find the values of : X and y



2

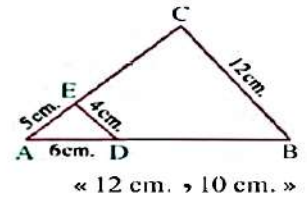
In the opposite figure :

$\triangle ADE \sim \triangle ABC$

Prove that : $\overline{DE} \parallel \overline{BC}$,

and from the lengths shown on the figure ,

find the length of each of : $\overline{BD}$ and $\overline{CE}$



3

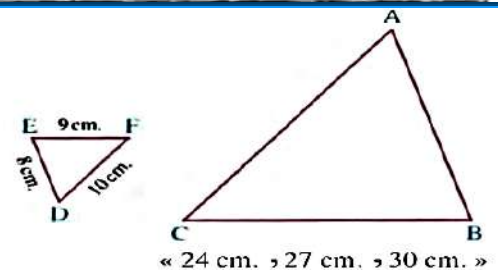
In the opposite figure :

$\triangle ABC \sim \triangle DEF$

, $DE = 8$ cm. , $EF = 9$ cm. , $FD = 10$ cm.

If the perimeter of $\triangle ABC = 81$ cm.

, find the side lengths of : $\triangle ABC$



4

Two similar rectangles , the dimensions of the first are 8 cm. and 12 cm. , and the perimeter of the second is 200 cm. Find the length of the second rectangle and its area.

« 60 cm. , 2400 cm². »

5

In the opposite figure :

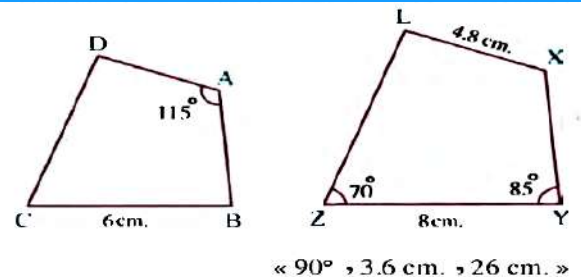
Polygon ABCD ~ polygon XYZL

(1) Calculate : $m(\angle XLZ)$, length of $\overline{AD}$

(2) If the perimeter of the polygon

ABCD = 19.5 cm.

Find : The perimeter of the polygon XYZL



6

If polygon ABCD ~ polygon XYZL , complete :

(1) $\frac{AB}{BC} = \frac{\dots\dots\dots}{YZ}$

(2) $AB \times ZL = XY \times \dots\dots\dots$

(3) $\frac{BC + YZ}{YZ} = \frac{\dots\dots\dots + LX}{LX}$

(4) $\frac{\text{perimeter of polygon } \dots\dots\dots}{\text{perimeter of polygon } \dots\dots\dots} = \frac{XY}{AB}$

7

The dimensions of a rectangle are 10 cm. and 6 cm. Find the perimeter and the area of another rectangle similar to it if :

(1) The scale factor equals 3

(2) The scale factor equals 0.4

SIMPLEST MATHS BOOKLETS

Lesson

2

Similarity of triangles

Cases of similarity of triangles

First case

Postulate (A. A. similarity postulate)

If two angles of one triangle are congruent to their two corresponding angles of another triangle, then the two triangles are similar.

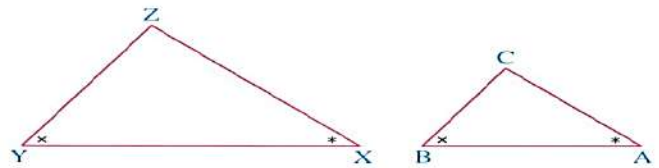
In the opposite figure :

If $\angle A \equiv \angle X$

, $\angle B \equiv \angle Y$

, then $\triangle ABC \sim \triangle XYZ$

and we deduce that : $\frac{AB}{XY} = \frac{BC}{YZ} = \frac{AC}{XZ}$



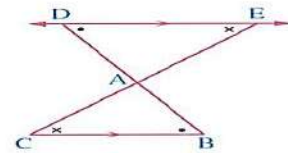
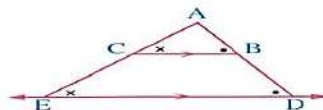
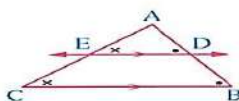
Remarks

- 1 The two right-angled triangles are similar if the measure of an acute angle in one of them equals the measure of an acute angle in the other.
- 2 The two isosceles triangles are similar if the measure of an angle in one of them equals the measure of the corresponding angle in the other.
- 3 Any two equilateral triangles are similar.

Corollary 1

If a line is drawn parallel to one side of a triangle and intersects the other two sides or the lines containing them, then the resulting triangle is similar to the original triangle.

In each of the following figures :



If $\overline{DE} \parallel \overline{BC}$ and intersects $\overline{AB}$ and $\overline{AC}$ at D and E respectively, then $\triangle ABC \sim \triangle ADE$

Corollary 2

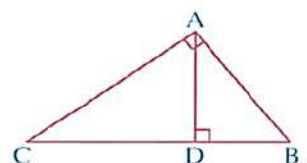
In any right-angled triangle, the altitude to the hypotenuse separates the triangle into two triangles which are similar to each other and to the original triangle.

In the opposite figure :

If $\triangle ABC$ is a right-angled triangle at A and $\overline{AD} \perp \overline{BC}$

, then $\triangle DBA \sim \triangle DAC \sim \triangle ABC$

and it is left to the student to prove this corollary by using the previous postulate and its remarks.



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Second case

Theorem 1 S.S.S. similarity theorem

If the side lengths of two triangles are in proportion , then the two triangles are similar.

Remark

For writing the two similar triangles in the same order of their corresponding vertices from the proportionality of their side lengths , we follow the following :

Let the vertices of one of the two triangles be A , B and C and the vertices of the other triangle be D , E and F and we have the proportion : $\frac{AC}{DF} = \frac{AB}{EF} = \frac{BC}{DE}$

We search for the vertices of the triangle which are opposite to the sides $\overline{AC}$, $\overline{AB}$ and $\overline{BC}$ respectively which are B , C and A

and we search for the vertices of the triangle which are opposite to the sides $\overline{DF}$, $\overline{EF}$ and $\overline{DE}$ respectively which are E , D and F , then :

$\Delta BCA \sim \Delta EDF$ or $\Delta ABC \sim \Delta FED$, etc ...

Third case

Theorem 2 S.A.S. similarity theorem

If an angle of one triangle is congruent to an angle of another triangle and lengths of the sides including those angles are in proportion , then the triangles are similar.



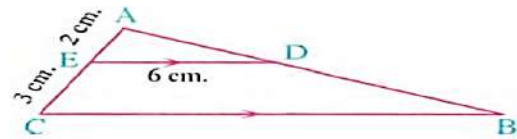
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Ex.1) Choose the correct answer :

(1) In the opposite figure :

If $\overline{ED} \parallel \overline{BC}$, $AE = 2$ cm.
 , $EC = 3$ cm. , $ED = 6$ cm.
 , then $BC = \dots\dots\dots$ cm.

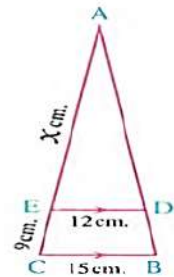
- (a) 9 (b) 15 (c) 12 (d) 10



(2) In the opposite figure :

$x = \dots\dots\dots$ cm.

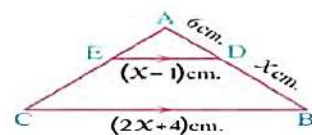
- (a) 12 (b) 24
 (c) 36 (d) 48



(3) In the opposite figure :

If $\overline{DE} \parallel \overline{BC}$, then $x = \dots\dots\dots$

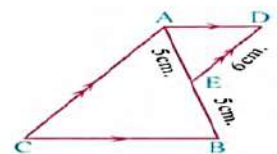
- (a) 10 (b) 30
 (c) 3 (d) 24



(4) In the opposite figure :

$AC = \dots\dots\dots$ cm.

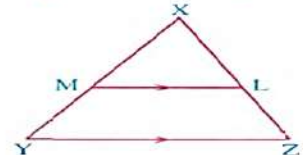
- (a) 6 (b) 9
 (c) 12 (d) 15



(5) In the opposite figure :

If $\overline{LM} \parallel \overline{YZ}$, $\frac{LM}{YZ} = \frac{4}{7}$
 , then $\frac{YM}{MX} = \dots\dots\dots$

- (a) $\frac{11}{4}$ (b) $\frac{3}{4}$
 (c) $\frac{4}{3}$ (d) $\frac{4}{11}$



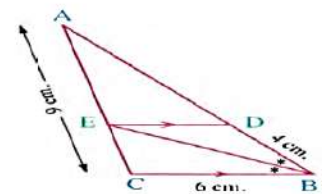
(6) In the opposite figure :

If $AC = 9$ cm. , $BD = 4$ cm.

, $BC = 6$ cm. ,

then the perimeter of $\triangle ADE = \dots\dots\dots$ cm.

- (a) 18 (b) 16
 (c) 14 (d) 12

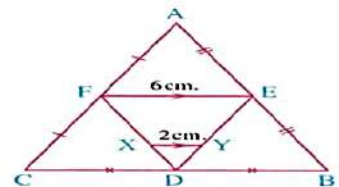


(7) In the opposite figure :

If the perimeter of $\triangle DXY = 8$ cm.

, then the perimeter of $\triangle ABC = \dots\dots\dots$ cm.

- (a) 18 (b) 24
 (c) 36 (d) 48



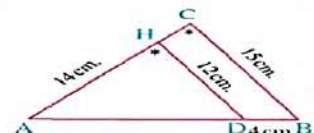
(8) In the opposite figure :

If $m(\angle AHD) = m(\angle C)$, $AH = 14$ cm. , $HD = 12$ cm.

, $CB = 15$ cm. , $DB = 4$ cm.

, then $AC + AD + AB = \dots\dots\dots$ cm.

- (a) 62.5 (b) 48 (c) 56



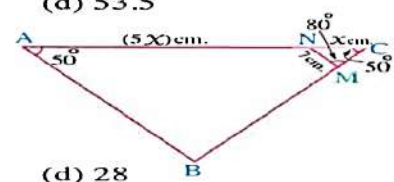
(9) In the opposite figure :

If $CN = x$ cm. , $NA = (5x)$ cm. , $MN = 7$ cm.

, $m(\angle C) = m(\angle A) = 50^\circ$, $m(\angle CMN) = 80^\circ$

, then $AB = \dots\dots\dots$ cm.

- (a) 21 (b) 35 (c) 42

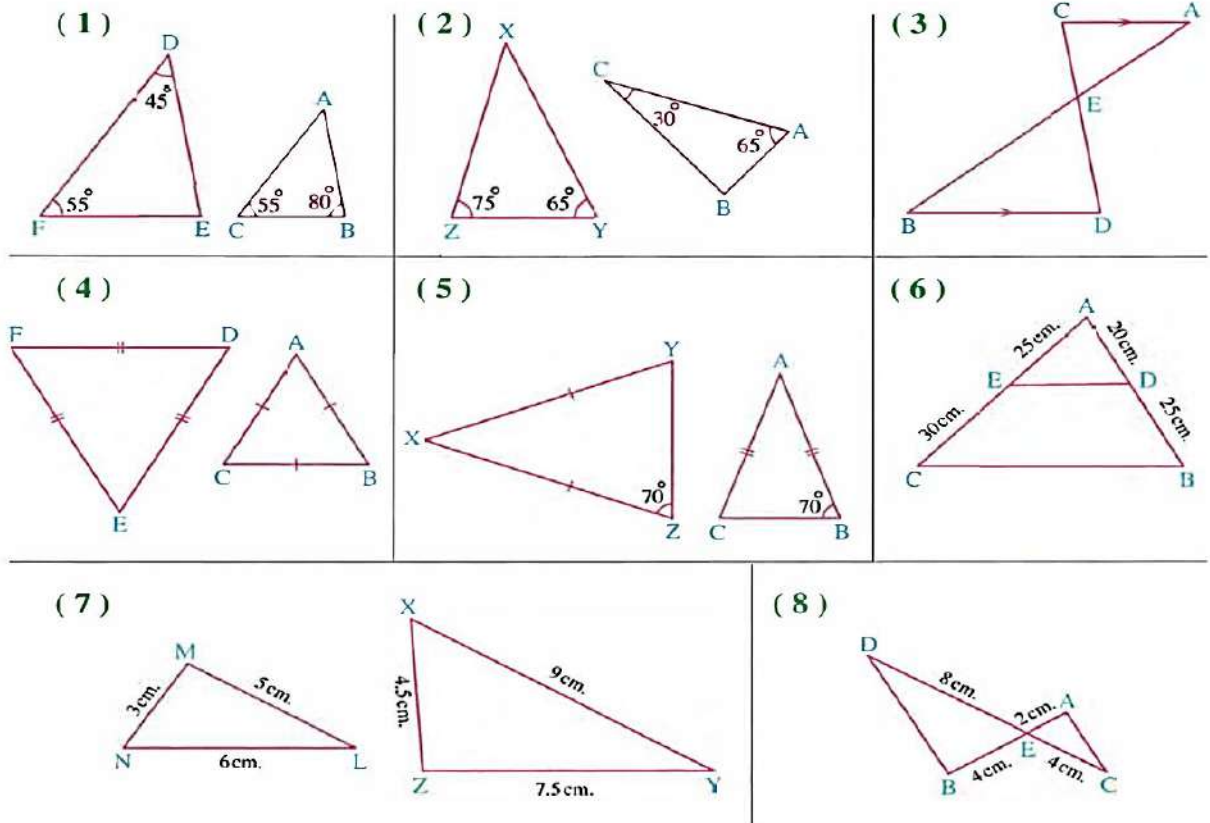


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Ex.2) Answer the following:

1

State in which of the following cases, the two triangles are similar. In case of similarity, state why they are similar :



2

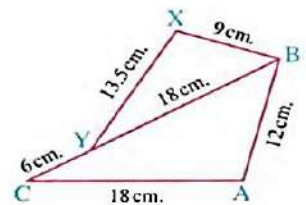
In the opposite figure :

B, Y and C are collinear.

Prove that :

(1) $\triangle XBY \sim \triangle ABC$

(2) $\overrightarrow{BC}$ bisects $\angle ABX$



3

In the opposite figure :

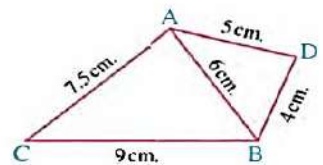
ABC is a triangle in which : $AB = 6 \text{ cm}$, $BC = 9 \text{ cm}$,

$AC = 7.5 \text{ cm}$, D is a point outside the triangle ABC where

$DB = 4 \text{ cm}$, $DA = 5 \text{ cm}$. Prove that :

(1) $\triangle ABC \sim \triangle DBA$

(2) $\overrightarrow{BA}$ bisects $\angle DBC$



4

In $\triangle ABC$, $AC > AB$, $M \in \overline{AC}$ where $m(\angle ABM) = m(\angle C)$

Prove that : $(AB)^2 = AM \times AC$

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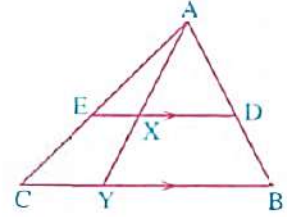
5

 In the opposite figure :

ABC is a triangle , $D \in \overline{AB}$, $\overrightarrow{DE} \parallel \overline{BC}$ and intersects $\overline{AC}$ at E ,
 $\overrightarrow{AX}$ is drawn to intersect $\overline{DE}$ and $\overline{BC}$ at X and Y respectively

(1) State three pairs of similar triangles.

(2) Prove that : $\frac{DX}{BY} = \frac{XE}{YC} = \frac{DE}{BC}$



6

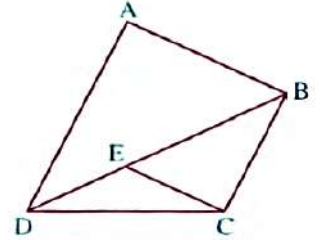
 In the opposite figure :

ABCD is a quadrilateral ,

$E \in \overline{BD}$ where $\frac{AB}{DA} = \frac{CE}{BC}$, $\frac{BD}{DA} = \frac{EB}{BC}$

Prove that : (1) $\overline{AD} \parallel \overline{BC}$

(2) $\overline{AB} \parallel \overline{CE}$



7

 $\overline{AB}$ and $\overline{DC}$ are two chords in a circle , $\overline{AB} \cap \overline{CD} = \{E\}$, where E lies outside the circle , $AB = 4$ cm. , $DC = 7$ cm. and $BE = 6$ cm.

Prove that : $\triangle ADE \sim \triangle CBE$, then find the length of : $\overline{CE}$

« 12 cm. »

8

 ABC is a right-angled triangle at A , $\overline{AD} \perp \overline{BC}$ to intersect it at D

If $\frac{BD}{DC} = \frac{1}{2}$ and $AD = 6\sqrt{2}$ cm.

, find the length of each of : $\overline{BD}$, $\overline{AB}$ and $\overline{AC}$

« 6 cm. , $6\sqrt{3}$ cm. , $6\sqrt{6}$ cm. »

9

 ABC and DEF are two similar triangles , $\overline{AX} \perp \overline{BC}$ to intersect it at X , $\overline{DY} \perp \overline{EF}$ to intersect it at Y Prove that : $BX \times YF = CX \times YE$

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SIMPLEST MATHS BOOKLETS

Lesson

3

The relation between the areas of two similar polygons

First The ratio between the areas of the surfaces of two similar triangles

Theorem 3

The ratio between the areas of the surfaces of two similar triangles equals the square of the ratio between the lengths of any two corresponding sides of the two triangles.

Remark 1

From the proof of the previous theorem we can deduce that :

The ratio between areas of two similar triangles equals the square of the ratio between two corresponding heights in them.

Example 1

If the ratio between the areas of two similar triangles is $\frac{9}{16}$, the perimeter of the smaller triangle is 60 cm.

Find : The perimeter of the greater triangle.

Solution

Let the two similar triangles be ΔABC , ΔXYZ where ΔABC is the smaller

$$\therefore \frac{a(\Delta ABC)}{a(\Delta XYZ)} = \left(\frac{AB}{XY}\right)^2 = \frac{9}{16}$$

$$\therefore \frac{AB}{XY} = \frac{3}{4}$$

$$\therefore \frac{\text{The perimeter of } \Delta ABC}{\text{The perimeter of } \Delta XYZ} = \frac{AB}{XY} = \frac{3}{4}$$

$$\therefore \frac{60}{\text{The perimeter of } \Delta XYZ} = \frac{3}{4}$$

$$\therefore \text{The perimeter of } \Delta XYZ = \frac{60 \times 4}{3} = 80 \text{ cm.}$$

(The req.)

Example 2

ABC is a triangle of area 62.5 cm^2 . Draw $\overline{XY} \parallel \overline{BC}$ to intersect $\overline{AB}$ at X and $\overline{AC}$ at Y

If $AX : XB = 2 : 3$

Find : The area of the figure XBCY

Solution

In ΔABC : $\therefore \overline{XY} \parallel \overline{BC}$

$$\therefore \Delta AXY \sim \Delta ABC$$

$$\therefore \frac{a(\Delta AXY)}{a(\Delta ABC)} = \left(\frac{AX}{AB}\right)^2$$

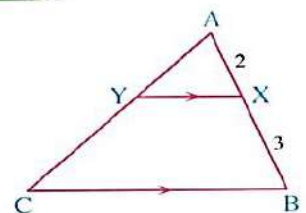
$$\therefore \frac{a(\Delta AXY)}{62.5} = \left(\frac{2}{5}\right)^2$$

$$\therefore a(\Delta AXY) = \frac{4}{25} \times 62.5 = 10 \text{ cm}^2$$

$$\therefore \text{The area of the figure XBCY} = a(\Delta ABC) - a(\Delta AXY)$$

$$= 62.5 - 10 = 52.5 \text{ cm}^2$$

(The req.)



SIMPLEST MATHS BOOKLETS

Remark 2

The ratio of the areas of the surfaces of two similar triangles equals the square of the ratio of the lengths of any two corresponding medians of the two triangles.

In the opposite figure :

If $\triangle ABC \sim \triangle DEF$, L is the midpoint of $\overline{BC}$, M is the midpoint of $\overline{EF}$

$$\therefore \frac{a(\triangle ABC)}{a(\triangle DEF)} = \left(\frac{AL}{DM} \right)^2$$

Proof :

$$\therefore \triangle ABC \sim \triangle DEF$$

$$\therefore BC = 2 BL, EF = 2 EM$$

$$\therefore \frac{AB}{DE} = \frac{BL}{EM}$$

$$\therefore \triangle ABL \sim \triangle DEM$$

$$\therefore \frac{a(\triangle ABC)}{a(\triangle DEF)} = \left(\frac{AB}{DE} \right)^2$$

$$\text{From (1), (2)} : \therefore \frac{a(\triangle ABC)}{a(\triangle DEF)} = \left(\frac{AL}{DM} \right)^2$$

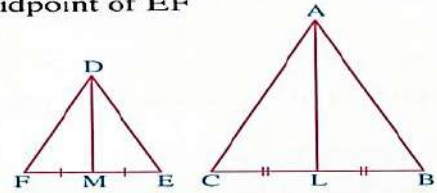
$$\therefore \frac{AB}{DE} = \frac{BC}{EF}$$

$$\therefore \frac{AB}{DE} = \frac{2 BL}{2 EM}$$

$$\therefore \angle B \equiv \angle E \quad (\text{Because } \triangle ABC \sim \triangle DEF)$$

$$\therefore \frac{a(\triangle ABL)}{a(\triangle DEM)} = \left(\frac{AB}{DE} \right)^2 = \left(\frac{AL}{DM} \right)^2 \quad (1)$$

$$(2)$$



Remark 3

In the opposite figure :

If $\triangle ABC \sim \triangle DEF$, $\overline{AN}$ bisects $\angle A$ and intersects $\overline{BC}$ at N

, $\overline{DZ}$ bisects $\angle D$ and intersects $\overline{EF}$ at Z

$$\therefore \frac{a(\triangle ABC)}{a(\triangle DEF)} = \left(\frac{AN}{DZ} \right)^2$$

Proof :

$$\therefore \triangle ABC \sim \triangle DEF$$

$$\therefore m(\angle BAC) = m(\angle EDF)$$

$$\therefore \frac{1}{2} m(\angle BAC) = \frac{1}{2} m(\angle EDF)$$

$$\therefore m(\angle BAN) = m(\angle EDZ)$$

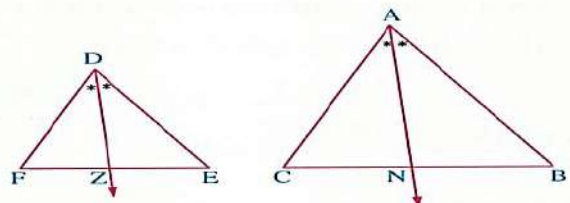
$$\therefore m(\angle B) = m(\angle E)$$

$$\therefore \triangle ABN \sim \triangle DEZ$$

$$\therefore \frac{a(\triangle ABN)}{a(\triangle DEZ)} = \left(\frac{AB}{DE} \right)^2 = \left(\frac{AN}{DZ} \right)^2 \quad (1)$$

$$\therefore \frac{a(\triangle ABC)}{a(\triangle DEF)} = \left(\frac{AB}{DE} \right)^2 \quad (2)$$

$$\text{From (1), (2)} : \therefore \frac{a(\triangle ABC)}{a(\triangle DEF)} = \left(\frac{AN}{DZ} \right)^2$$



Remark 4

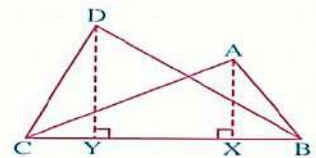
The ratio of the areas of two triangles having a common base equals the ratio of the two heights of the two triangles.

In the opposite figure :

$\overline{BC}$ is a common base of $\triangle ABC$, $\triangle DBC$

$$\therefore \frac{a(\triangle ABC)}{a(\triangle DBC)} = \frac{\frac{1}{2} BC \times AX}{\frac{1}{2} BC \times DY} = \frac{AX}{DY}$$

Notice that : It is not necessary that the two triangles are similar.



Remark 5

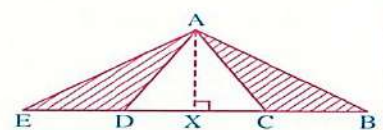
The ratio of the areas of two triangles having a common height equals the ratio of the lengths of two bases of the two triangles.

In the opposite figure :

AX is a common height for $\triangle ABC$, $\triangle ADE$

$$\therefore \frac{a(\triangle ABC)}{a(\triangle ADE)} = \frac{\frac{1}{2} BC \times AX}{\frac{1}{2} DE \times AX} = \frac{BC}{DE}$$

Notice that : It is not necessary that the two triangles are similar.



SIMPLEST MATHS BOOKLETS

Second The ratio between the areas of the surfaces of two similar polygons

Fact

Any two similar polygons can be divided into the same number of triangles, each is similar to its corresponding one.

Remarks

- The previous fact is correct whatever the number of sides of the two similar polygons (having always the same number of sides)
- If the number of sides of a polygon is n sides, then the number of the triangles that the polygon is divided by drawing the diagonals from one of its vertices = $(n - 2)$ triangles

Theorem 4

The ratio between the areas of the surfaces of two similar polygons equals the square of the ratio between the lengths of any two corresponding sides of the polygons.

Ex.1) Choose the correct answer :

- (1) The ratio between the perimeters of two similar polygons is $4 : 9$, so the ratio between their areas is
- (a) $4 : 9$ (b) $9 : 4$ (c) $2 : 3$ (d) $16 : 81$
- (2)  If $\Delta ABC \sim \Delta XYZ$, $AB = 3 XY$, then $\frac{a(\Delta XYZ)}{a(\Delta ABC)} = \dots\dots\dots$
- (a) 3 (b) 9 (c) $\frac{1}{3}$ (d) $\frac{1}{9}$
- (3) If the ratio between the areas of two similar polygons is $9 : 49$, then the ratio between the lengths of their two corresponding sides is
- (a) $3 : 7$ (b) $9 : 49$ (c) $3 : 10$ (d) $10 : 3$
- (4) The ratio between the corresponding sides of two similar triangle is $2 : 5$, if the area of the first one is 16 cm^2 , then the area of the second one = cm^2 .
- (a) 40 (b) 80 (c) 100 (d) 120
- (5)  If the lengths of two corresponding sides in two similar polygons are 12 cm. , 16 cm. and the area of the smaller polygon = 135 cm^2 , then the area of the greater polygon cm^2
- (a) 24 (b) 180 (c) 240 (d) 200

SIMPLEST MATHS BOOKLETS

- (6) If the ratio between perimeters of two similar polygon is 5 : 7 and the area of the greater polygon is 245 cm^2 , then the area of the smaller polygon equals cm^2
 (a) 125 (b) 175 (c) 343 (d) 480.2
- (7) The ratio between two corresponding sides of two similar squares is 3 : 4 , if the area of the greater square is 48 cm^2 , then the area of the smaller one = cm^2
 (a) 16 (b) 12 (c) 20 (d) 27
- (8) The ratio between the lengths of the diagonals of two squares is 2 : 5 , if the area of the smaller one is 4 cm^2 , so the area of the greater one is cm^2
 (a) 25 (b) 16 (c) 10 (d) 20
- (9) If the ratio between areas of two similar triangles equals 9 : 25 and the perimeter of the smaller triangle is 60 cm. , then the perimeter of the greater triangle equals
 (a) 60 (b) 80 (c) 100 (d) 120
- (10)  If $\Delta ABC \sim \Delta DEF$, a $(\Delta ABC) = 9$ a (ΔDEF) and $DE = 4 \text{ cm.}$, then $AB = \dots\dots\dots \text{cm.}$
 (a) $\frac{4}{3}$ (b) 12 (c) 9 (d) 36
- (11) The ratio between the diameters of two circles is 3 : 5 , if the area of the inscribed square in the smaller circle is 27 cm^2 , then the area of the inscribed square in the greater circle equals cm^2
 (a) 45 (b) 50 (c) 75 (d) 100
- (12) The ratio between two corresponding sides of two similar polygons is 3 : 4 , if the sum of its two areas is 150 cm^2 , then the area of the smaller polygon = cm^2
 (a) 54 (b) 96 (c) 75 (d) 52
- (13) The ratio between the lengths of two corresponding sides in two similar polygons is 5 : 3 and the difference between their areas is 32 cm^2 , then the area of the smaller polygon is cm^2
 (a) 18 (b) 50 (c) 32 (d) 16
- (14) If the polygon $M_1 \sim$ the polygon M_2 and $\frac{\text{area of polygon } M_1}{\text{area of polygon } M_2} = \frac{9}{16}$, then it means that
 (a) the sum of their areas = 25 square units.
 (b) the ratio between the two corresponding sides = 9 : 16
 (c) the scale factor of the similarity of M_1 to $M_2 = \frac{9}{16}$
 (d) the perimeter of polygon $M_1 = \frac{3}{4}$ the perimeter of polygon M_2

SIMPLEST MATHS BOOKLETS

(15) If the polygon $ABCD \sim$ the polygon $\hat{A}\hat{B}\hat{C}\hat{D}$, $\frac{AB}{\hat{A}\hat{B}} = \frac{1}{3}$

, then $\frac{a(\text{the polygon } ABCD)}{a(\text{the polygon } \hat{A}\hat{B}\hat{C}\hat{D})} + \frac{\text{perimeter of } (ABCD)}{\text{perimeter of } (\hat{A}\hat{B}\hat{C}\hat{D})} = \dots\dots\dots$

(a) $\frac{2}{3}$

(b) $\frac{4}{5}$

(c) $\frac{5}{9}$

(d) $\frac{4}{9}$

(16) In the opposite figure :

If $AB = 3$ cm. , $BE = 5$ cm. , $ED = 7$ cm.

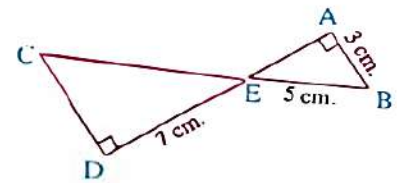
, then $\frac{a(\Delta ABE)}{a(\Delta CDE)} \times \frac{m(\angle ABE)}{m(\angle DCE)} = \dots\dots\dots$

(a) $\frac{9}{49}$

(b) $\frac{25}{49}$

(c) $\frac{9}{25}$

(d) $\frac{16}{49}$



(17) In the opposite figure :

If $\overline{DE} \parallel \overline{BC}$, $DE = 4$ cm. , $BC = 9$ cm.

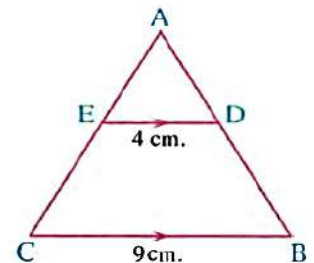
, then $\frac{a(\Delta ADE)}{a(\Delta ABC)} = \dots\dots\dots$

(a) $\frac{16}{81}$

(b) $\frac{81}{65}$

(c) $\frac{65}{81}$

(d) $\frac{16}{65}$



(18) In the opposite figure :

If $AX : XB = 5 : 3$, $a(\Delta ABC) = 25.6$ cm²

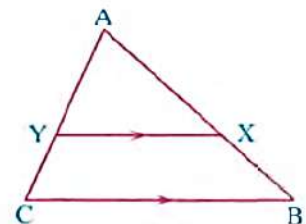
, then $a(\Delta AXY) = \dots\dots\dots$ cm²

(a) 10

(b) 16

(c) 41

(d) 65.5



(19) In the opposite figure :

If $\overline{BE} \parallel \overline{DC}$

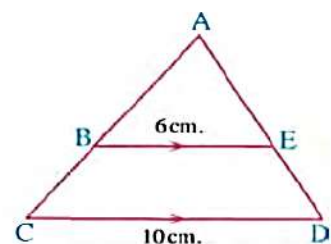
, then $\frac{\text{the area of } \Delta ABE}{\text{the area of trapezium BCDE}} = \dots\dots\dots$

(a) $\frac{25}{81}$

(b) $\frac{3}{5}$

(c) $\frac{9}{16}$

(d) $\frac{9}{25}$



(20) In the opposite figure :

$\overline{DE} \parallel \overline{BC}$, the area of $\Delta ADE = 8$ cm²

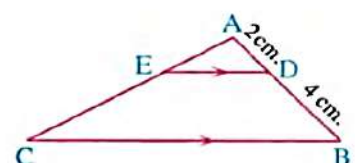
, then the area of the figure DBCE = $\dots\dots\dots$ cm²

(a) 27

(b) 64

(c) 24

(d) 16



SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

1 The ratio between the two perimeters of two similar triangles is 3 : 2 and the sum of their areas is 130 cm^2 . Find the area of each of them. « $90 \text{ cm}^2, 40 \text{ cm}^2$ »

2 The ratio between the lengths of two corresponding sides in two similar polygons is 1 : 3. Let the difference between their areas be 32 cm^2 , so find the area of each. « $4 \text{ cm}^2, 36 \text{ cm}^2$ »

3 $\square$ ABC is a triangle, $D \in \overline{AB}$ where $AD = 2 BD$, $E \in \overline{AC}$ where $\overline{DE} \parallel \overline{BC}$. If the area of $\triangle ADE = 60 \text{ cm}^2$, find the area of the trapezium DBCE « 75 cm^2 »

4 $\square$ In the opposite figure :

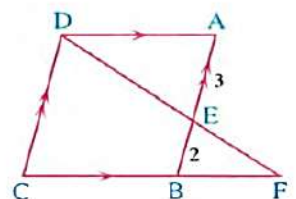
ABCD is a parallelogram

, $E \in \overline{AB}$ where $\frac{AE}{EB} = \frac{3}{2}$

, $\overrightarrow{DE} \cap \overrightarrow{CB} = \{F\}$

(1) Prove that : $\triangle DCF \sim \triangle EAD$

(2) Find : $\frac{a(\triangle DCF)}{a(\triangle EAD)}$



« $\frac{25}{9}$ »

5 $\square$ ABCD is a parallelogram, $X \in \overline{AB}$, $X \notin \overline{AB}$ where $BX = 2 AB$, $Y \in \overline{CB}$, $Y \notin \overline{CB}$ where $BY = 2 BC$, the parallelogram BXZY is drawn.

Prove that : $\frac{a(\text{parallelogram ABCD})}{a(\text{parallelogram XBYZ})} = \frac{1}{4}$

6 $\square$ ABCD, XYZL are two similar polygons. If M is the midpoint of $\overline{BC}$ and N is the midpoint of $\overline{YZ}$, prove that : $a(\text{polygon ABCD}) : a(\text{polygon XYZL}) = (MD)^2 : (NL)^2$

7 $\square$ ABC is a right-angled triangle at B. The equilateral triangles ABX, BCY, ACZ are drawn. Prove that : $a(\triangle ABX) + a(\triangle BCY) = a(\triangle ACZ)$

8 $\square$ ABC is an inscribed triangle in a circle where $\frac{AB}{BC} = \frac{4}{3}$, from B a tangent is drawn to the circle to intersect $\overline{AC}$ at E

Prove that : $\frac{a(\triangle ABC)}{a(\triangle ABE)} = \frac{7}{16}$

9 $\square$ ABC is a right-angled triangle at B, $\overline{BD} \perp \overline{AC}$ to intersect it at D. The squares AXYB, BMNC are drawn on $\overline{AB}$, $\overline{BC}$ respectively outside the triangle ABC

(1) Prove that : The polygon DAXYB $\sim$ the polygon DBMNC

(2) If $AB = 6 \text{ cm}$, $AC = 10 \text{ cm}$.

, find : the ratio between areas of the two polygons.

« $\frac{9}{16}$ »

SIMPLEST MATHS BOOKLETS

Lesson

4

Applications of similarity in the circle

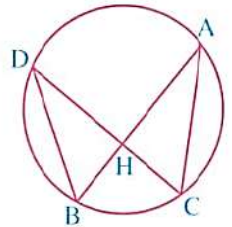
1 In the opposite figure :

$\overline{AB}$, $\overline{CD}$ are two intersecting chords at H

We notice that : $\triangle HAC \sim \triangle HDB$

because : $m(\angle AHC) = m(\angle DHB)$ (V.O.A)

, $m(\angle A) = m(\angle D)$ (two inscribed angles subtended by the same arc $\widehat{CB}$)



► From similarity , we deduce that :

$$\frac{HA}{HD} = \frac{HC}{HB} \quad \therefore HA \times HB = HC \times HD$$

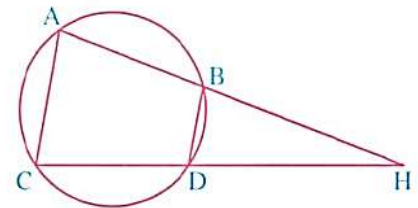
2 In the opposite figure :

ABCD is a cyclic quadrilateral , $\overrightarrow{AB} \cap \overrightarrow{CD} = \{H\}$

We notice that : $\triangle HAC \sim \triangle HDB$

because : $m(\angle HAC) = m(\angle HDB)$ (properties of cyclic quadrilateral)

, $\angle H$ is a common angle.



► From similarity , we deduce that :

$$\frac{HA}{HD} = \frac{HC}{HB} \quad \therefore HA \times HB = HC \times HD$$

Well known problem

If the two lines containing the two chords $\overline{AB}$, $\overline{CD}$ of a circle intersect at the point E

, then $EA \times EB = EC \times ED$

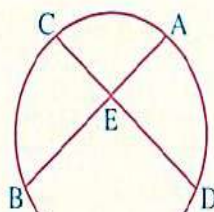


Fig. (1)

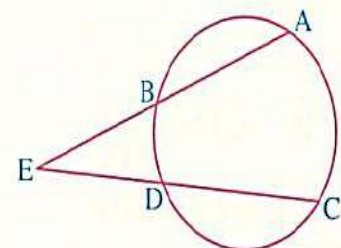


Fig. (2)

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Remark

In the opposite figure :

$\overline{AB}$ is a tangent to the circle at B

We notice that : $\triangle ABC \sim \triangle ADB$

This is because : $m(\angle ABC) = m(\angle D)$

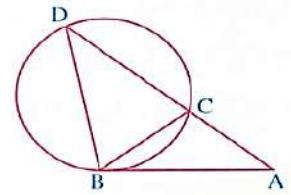
(tangency and inscribed angles subtended by $\widehat{BC}$)

, $\angle A$ is a common angle

From similarity we deduce that :

$$\frac{AB}{AD} = \frac{AC}{AB}$$

$$\therefore (AB)^2 = AC \times AD$$



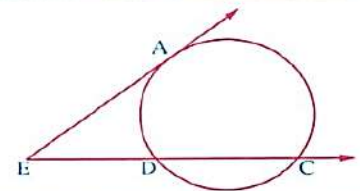
Remember that

AB is a mean proportion of AC , AD

Corollary 1

If E is a point outside the circle , $\overrightarrow{EA}$ is a tangent to the circle at A , $\overrightarrow{EC}$ intersects it at D , C , then

$$(EA)^2 = ED \times EC$$



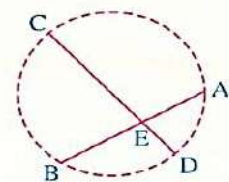
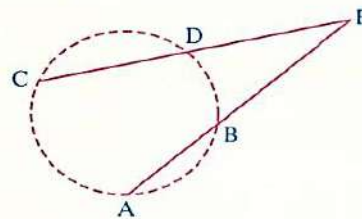
Converse of the well known problem

If the two lines containing the two segments $\overline{AB}$ and $\overline{CD}$ intersect at the point E (A , B , C , D and E are distinct points) and $EA \times EB = EC \times ED$, then the points A , B , C and D lie on a circle.

In the opposite figures :

If $EA \times EB = EC \times ED$

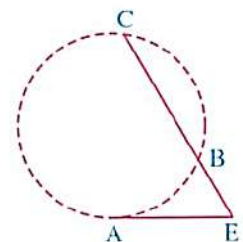
, then the points A , B , C and D lie on the same circle.



Corollary 2

If $(EA)^2 = EB \times EC$

, then $\overline{EA}$ is a tangent segment to the circle which passes through the points A , B and C

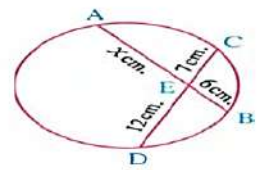


SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

$x = \dots\dots\dots$ cm.

- (a) 3.5 (b) 14
(c) 6 (d) 12



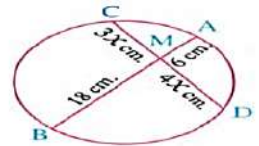
(2) In the opposite figure :

$\overline{AB} \cap \overline{CD} = \{M\}$, $AM = 6$ cm.

, $MB = 18$ cm. , $CM = 3x$ cm.

, $DM = 4x$ cm. , then $CD = \dots\dots\dots$ cm.

- (a) 3 (b) 9
(c) 18 (d) 21



(3) In the opposite figure :

$x = \dots\dots\dots$

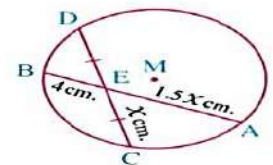
- (a) 6 (b) -6
(c) ± 6 (d) 36



(4) In the opposite figure :

$x = \dots\dots\dots$ cm.

- (a) 6.5 (b) 13
(c) 6 (d) 36



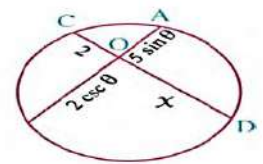
(5) In the opposite figure :

If $\overline{AB}$, $\overline{CD}$ are two chords in the circle ,

$\overline{AB} \cap \overline{CD} = \{O\}$, $AO = (5 \sin \theta)$ cm.

, $OB = (2 \csc \theta)$ cm. , $OC = 2$ cm. , then $x = \dots\dots\dots$ cm.

- (a) 5 (b) 10 (c) $\frac{\sqrt{3}}{2}$ (d) $10\sqrt{3}$



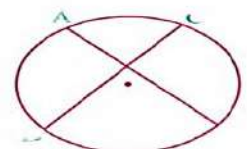
(6) In the opposite figure :

$\overline{AB} \cap \overline{CD} = \{E\}$, $AE = 4$ cm.

, $EB = 6$ cm. , $DE = (x + 1)$ cm.

, $CE = (x - 1)$ cm. , then $x = \dots\dots\dots$ cm.

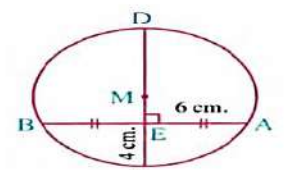
- (a) 5 (b) 6
(c) 4 (d) 7



(7) In the opposite figure :

The radius length of the circle = $\dots\dots\dots$ cm.

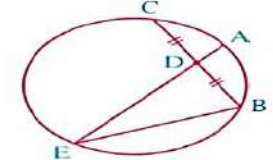
- (a) 9 (b) 4.5
(c) 6 (d) 6.5



(8) In the opposite figure :

$(BD)^2 = \dots\dots\dots$

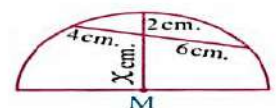
- (a) $AD \times DB$ (b) $AD \times DE$
(c) $AD \times BE$ (d) $AC \times BD$



(9) In the opposite figure :

A semicircle of centre M , then $x = \dots\dots\dots$ cm.

- (a) 5 (b) 7 (c) 8 (d) 12

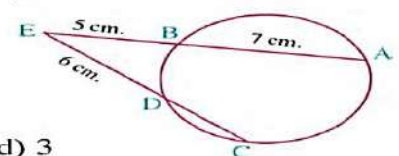


(10) In the opposite figure :

If $AB = 7$ cm. , $BE = 5$ cm. , $DE = 6$ cm.

, then the length of $\overline{CD} = \dots\dots\dots$ cm.

- (a) 6 (b) 5 (c) 4 (d) 3

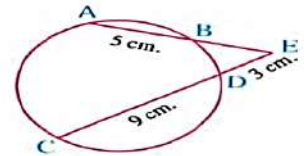


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(11) In the opposite figure :

BE = cm.

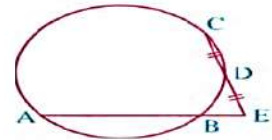
- (a) 6 (b) 5 (c) 4 (d) 3



(12) In the opposite figure :

If $DE = DC$, $EB = 2$ cm. , $AB = 7$ cm.
 , then the length of $\overline{EC}$ = cm.

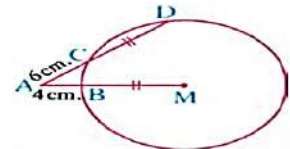
- (a) 6 (b) 4
(c) 5 (d) 3



(13) In the opposite figure :

If $DC = MB$, then the circumference of circle M = cm.

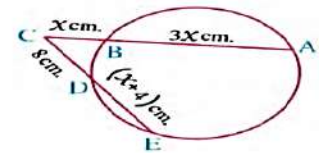
- (a) 15π (b) 18π
(c) 20π (d) 24π



(14) In the opposite figure :

x =

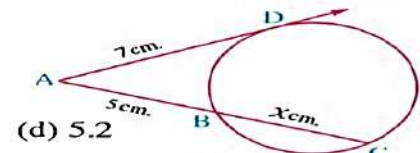
- (a) 5 (b) 6
(c) 3 (d) 9



(15) In the opposite figure :

x =

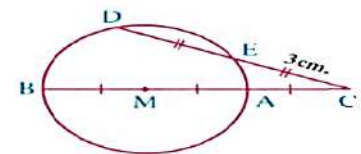
- (a) 4.8 (b) 5.6 (c) 4.2



(16) In the opposite figure :

The area of the circle M = cm^2

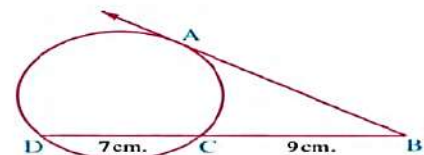
- (a) 6π (b) 18π
(c) $2\sqrt{6}\pi$ (d) $\sqrt{6}\pi$



(17) In the opposite figure :

$\overline{BA}$ is a tangent , $BC = 9$ cm. , $CD = 7$ cm.
 , then AB = cm.

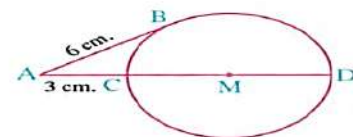
- (a) 63 (b) 144
(c) 12 (d) $\frac{9}{16}$



(18) In the opposite figure :

If $\overline{AB}$ is a tangent segment to circle M
 , then the circumference of circle M =

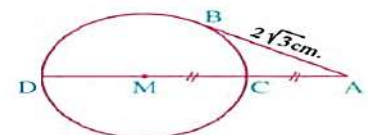
- (a) 6π (b) 9π
(c) 12π (d) 15π



(19) In the opposite figure :

The length of the radius of circle M = cm.

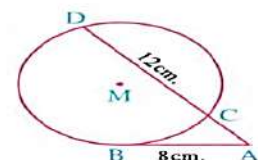
- (a) 2 (b) 3
(c) 4 (d) 5



(20) In the opposite figure :

AC = cm.

- (a) 12 (b) 18
(c) 4 (d) 6



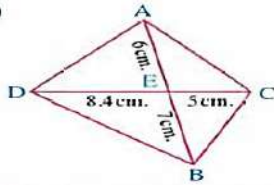
SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

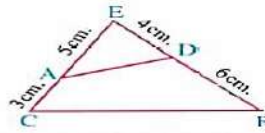
1

In which of the following figures , the points A , B , C and D lie on a circle ? Explain your answer.

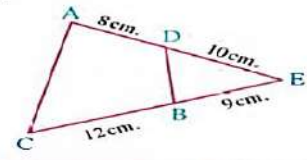
(1)



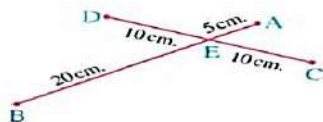
(2)



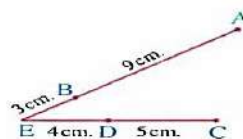
(3)



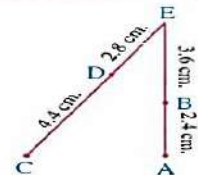
(4)



(5)



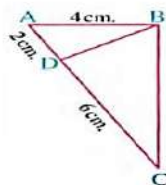
(6)



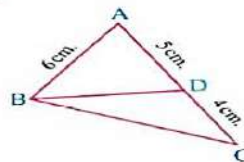
2

In which of the following figures , $\overline{AB}$ is a tangent segment to the circle which passes through the points B , C and D ?

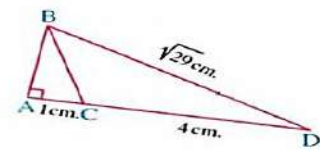
(1)



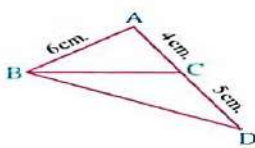
(2)



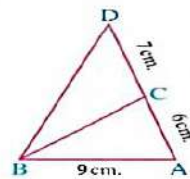
(3)



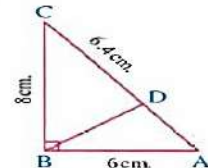
(4)



(5)



(6)



3

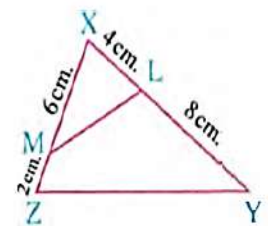
In the opposite figure :

$L \in \overline{XY}$ where $XL = 4$ cm. ,

$YL = 8$ cm. , $M \in \overline{XZ}$

where $XM = 6$ cm. , $ZM = 2$ cm.

Prove that : (1) $\triangle XLM \sim \triangle XZY$ (2) $LYZM$ is a cyclic quadrilateral.



4

$\overline{AB} \cap \overline{CD} = \{E\}$, $AE = \frac{5}{12} BE$, $DE = \frac{3}{5} EC$ If $BE = 6$ cm. and $CE = 5$ cm.

Prove that : The points A , B , C and D lie on one circle.

SIMPLEST MATHS BOOKLETS

5

Two circles are intersecting at A and B , $C \in \overleftrightarrow{AB}$ and $C \notin \overline{AB}$, from C the two tangent segments $\overline{CX}$ and $\overline{CY}$ are drawn to touch the circles at X and Y respectively.

Prove that : $CX = CY$

6

In the opposite figure :

M and N are two circles touching externally at E

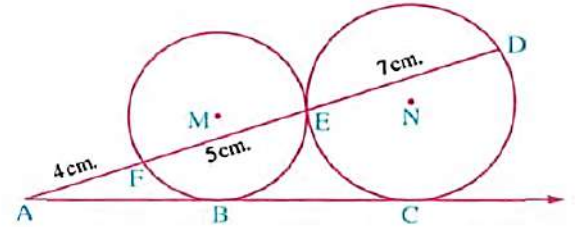
, $\overline{AC}$ touches the circle M at B and touches

the circle N at C , $\overline{AE}$ intersects the two

circles at F and D respectively ,

where $AF = 4$ cm. , $FE = 5$ cm. , $ED = 7$ cm.

Prove that : B is the midpoint of $\overline{AC}$



7

ABC is a triangle $D \in \overline{BC}$ where $BD = 5$ cm. and $DC = 4$ cm. If $AC = 6$ cm.

, prove that :

(1) $\overline{AC}$ is a tangent segment to the circle passing through the points A , B and D

(2) $\triangle ACD \sim \triangle BCA$

(3) Area of $(\triangle ABD)$: area of $(\triangle ABC) = 5 : 9$

8

Two concentric circles at M , the lengths of their radii are 12 cm. and 7 cm.

$\overline{AD}$ is a chord in the larger circle to intersect the smaller circle at B and C respectively.

Prove that : $AB \times BD = 95$

9

ABCD is a rectangle in which $AB = 6$ cm. and $BC = 8$ cm. , $\overline{BE} \perp \overline{AC}$ and intersects $\overline{AC}$ at E and $\overline{AD}$ at F

(1) **Prove that :** $(AB)^2 = AF \times AD$

(2) **Find the length of :** $\overline{AF}$

« 4.5 cm. »

Unit 4

TRIANGLE PROPORTIONALITY THEOREMS



Parallel Lines and
Proportional Parts



Thales' Theorem



Angle Bisector and
Proportional Parts



Converse of the
Angle Bisector Theorem

SIMPLEST MATHS BOOKLETS

Lesson

1

Parallel lines and proportional parts

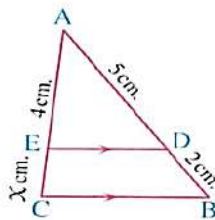
Theorem 1

If a line is drawn parallel to one side of a triangle and intersects the other two sides, then it divides them into segments whose lengths are proportional.

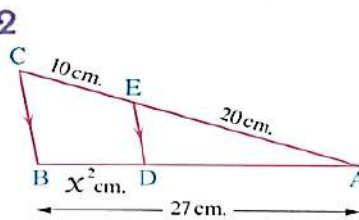
Example 1

In each of the following figures : $\overline{DE} \parallel \overline{BC}$ Find the value of x

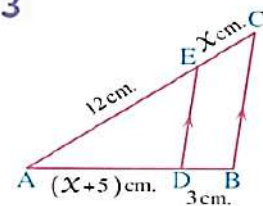
1



2



3



Solution

1 $\therefore \overline{DE} \parallel \overline{BC}$

$$\therefore \frac{5}{2} = \frac{4}{x}$$

2 $\therefore \overline{DE} \parallel \overline{BC}$

$$\therefore x^2 = 9$$

3 $\therefore \overline{DE} \parallel \overline{BC}$

$$\therefore x^2 + 5x = 36$$

$$\therefore (x+9)(x-4) = 0$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\therefore x = 1.6$$

$$\therefore \frac{AB}{DB} = \frac{AC}{EC}$$

$$\therefore x = \pm 3$$

$$\therefore \frac{AE}{EC} = \frac{AD}{DB}$$

$$\therefore x^2 + 5x - 36 = 0$$

$$\therefore x = -9 \text{ (refused) or } x = 4$$

$$\therefore \frac{27}{x^2} = \frac{30}{10}$$

$$\therefore \frac{12}{x} = \frac{x+5}{3}$$

Corollary

If a straight line is drawn outside the triangle ABC parallel to one side of its sides, say $\overline{BC}$ intersecting $\overline{AB}$ and $\overline{AC}$ at D and E respectively, as shown in the figures, then $\frac{AB}{BD} = \frac{AC}{CE}$

Converse of theorem 1

If a straight line intersects two sides of a triangle and divides them into segments whose lengths are proportional, then it is parallel to the third side of the triangle.

SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

(1) In the opposite figure :

First : If $\frac{AD}{DB} = \frac{5}{3}$, then $\frac{AB}{BD} = \dots\dots\dots$

(a) $\frac{3}{5}$

(b) $\frac{8}{3}$

(c) $\frac{3}{8}$

(d) $\frac{5}{8}$

Second : If $\frac{AE}{AC} = \frac{4}{7}$, then $\frac{CE}{EA} = \dots\dots\dots$

(a) $\frac{7}{4}$

(b) $\frac{4}{3}$

(c) $\frac{2}{5}$

(d) $\frac{3}{4}$

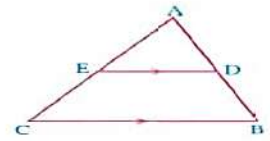
Thirld : If $\frac{DE}{BC} = \frac{3}{5}$, then $\frac{AD}{DB} = \dots\dots\dots$

(a) $\frac{5}{3}$

(b) 1.5

(c) $\frac{2}{3}$

(d) $\frac{3}{4}$



(2) In the opposite figure :

If $\overline{DE} \parallel \overline{BC}$, $AD = 2$ cm.

and $AE = DB = 3$ cm.

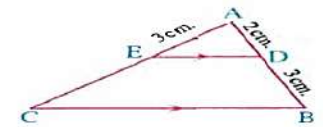
, then the length of $\overline{EC} = \dots\dots\dots$ cm.

(a) 3

(b) 4

(c) 5

(d) 4.5



(3) In the opposite figure :

$\overline{AB} \parallel \overline{DE}$, $\overline{AE} \cap \overline{BD} = \{C\}$

, $AC = 6$ cm. , $BC = 4$ cm. and $CD = 3$ cm.

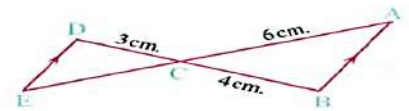
, then the length of $\overline{CE} = \dots\dots\dots$ cm.

(a) 5

(b) 4

(c) 4.5

(d) 3.5



(4) In the opposite figure :

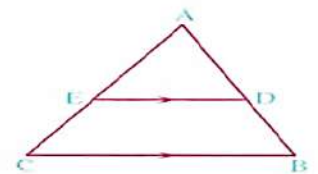
All the following statements are true except

(a) $\frac{AD}{DB} = \frac{AE}{EC}$

(b) $\frac{AD}{DB} = \frac{DE}{BC}$

(c) $\frac{AD}{AB} = \frac{AE}{AC}$

(d) $\frac{AB}{BD} = \frac{AC}{EC}$



(5) In the opposite figure :

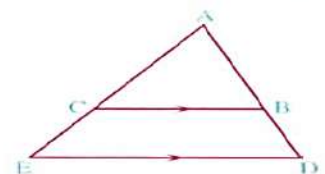
If $\overline{BC} \parallel \overline{DE}$, then

(a) the shape DBCE is a cyclic quadrilateral

(b) $\triangle ABC \sim \triangle ADE$

(c) $AB \times AD = AC \times AE$

(d) $\frac{AB}{BD} = \frac{BC}{DE}$



(6) In the opposite figure :

If $\overline{DE} \parallel \overline{AC}$, $BE = 3$ cm. , $EC = 2$ cm.

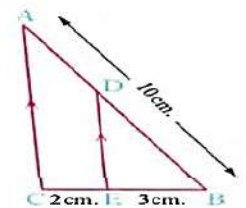
, then $AD = \dots\dots\dots$ cm.

(a) 6

(b) 4

(c) 5

(d) 7



(7) In the opposite figure :

If $\overline{XY} \parallel \overline{BC}$, $\frac{AX + AY}{AB + AC} = \frac{3}{5}$

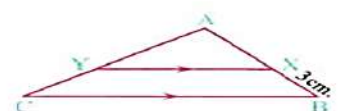
, then $AX = \dots\dots\dots$ cm.

(a) 3

(b) 6

(c) 4.5

(d) 4



(8) In the opposite figure :

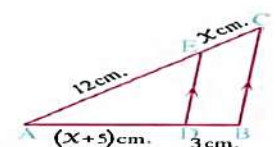
$\overline{DE} \parallel \overline{BC}$, then $x = \dots\dots\dots$

(a) 4

(b) 9

(c) 12

(d) 3



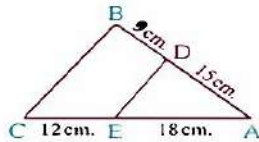
SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

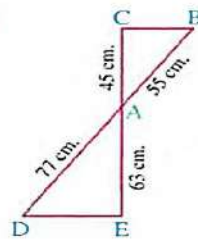
1

In each of the following figures, is $\overline{DE} \parallel \overline{BC}$?

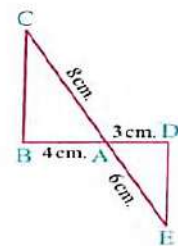
(1)



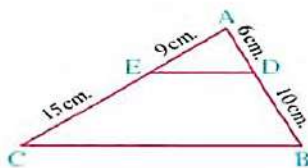
(2)



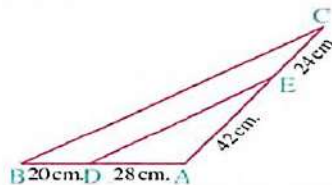
(3)



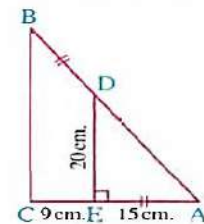
(4)



(5)



(6)



2

$\overline{XY} \cap \overline{ZL} = \{M\}$, where $\overline{XZ} \parallel \overline{LY}$, if $XM = 9$ cm, $YM = 15$ cm, and $ZL = 36$ cm, find the length of : $\overline{ZM}$

« 13.5 cm. »

3

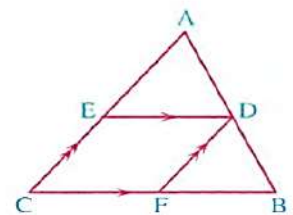
For each of the following, use the opposite figure and the given data to find the value of x (Lengths are measured in centimetres) :

(1) $AD = 4$, $BD = 8$, $CE = 6$ and $AE = x$

(2) $AE = x$, $EC = 5$, $AD = x - 2$ and $DB = 3$

(3) $AB = 21$, $BF = 8$, $FC = 6$ and $AD = x$

(4) $AD = x$, $BF = x + 5$ and $2 DB = 3 FC = 12$



4

XYZ is a triangle in which $XY = 14$ cm, $XZ = 21$ cm, $L \in \overline{XY}$, where $XL = 5.6$ cm, and $M \in \overline{XZ}$ where $XM = 8.4$ cm. **Prove that :** $\overline{LM} \parallel \overline{YZ}$

5

In the triangle ABC , $D \in \overline{AB}$, $E \in \overline{AC}$ and $5 AE = 4 EC$. If $AD = 10$ cm, and $DB = 8$ cm, is $\overline{DE} \parallel \overline{BC}$? Explain your answer.

6

$ABCD$ is a quadrilateral, its diagonals are intersected at E . If $AE = 6$ cm, $BE = 13$ cm, $EC = 10$ cm, and $ED = 7.8$ cm, **prove that :** $ABCD$ is a trapezium.

7

Prove that : The line segment drawn between two midpoints of two sides in a triangle is parallel to the third side and its length is equal to a half of the length of this side.

SIMPLEST MATHS BOOKLETS

8  ABC is a triangle , $D \in \overline{AB}$, where $3 AD = 2 DB$ and $E \in \overline{AC}$, where $5 CE = 3 AC$ and $\overline{AX}$ is drawn to intersect $\overline{BC}$ at X , if $AF = 8$ cm. and $AX = 20$ cm. where $F \in \overline{AX}$
Prove that : The points D , F and E are collinear.

9  ABC is a triangle , $D \in \overline{BC}$, where $\frac{BD}{DC} = \frac{3}{4}$ and $E \in \overline{AD}$, where $\frac{AE}{AD} = \frac{3}{7}$, $\overline{CE}$ is drawn to intersect $\overline{AB}$ at X , $\overline{DY} \parallel \overline{CX}$ and intersects $\overline{AB}$ at Y **Prove that :** $AX = BY$

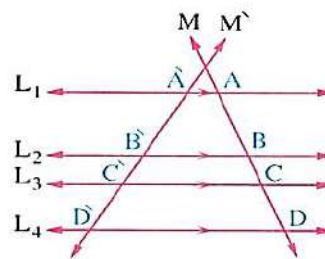
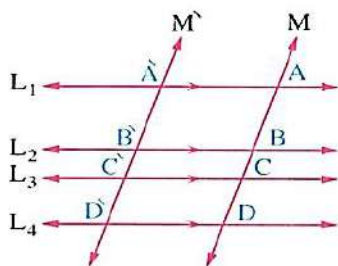
Lesson

2

Talis' theorem

Theorem 2

Given several coplanar parallel lines and two transversals , then the lengths of the corresponding segments on the transversals are proportional.

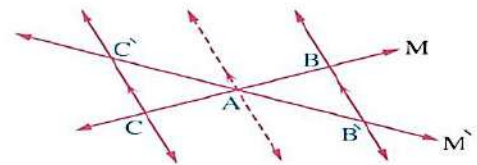


In the above two figures :

If $L_1 \parallel L_2 \parallel L_3 \parallel L_4$ and $M, \vec{M}$ are two transversals , then $\frac{AB}{A'B'} = \frac{BC}{B'C'} = \frac{CD}{C'D'} = \frac{AC}{A'C'}$

Two special cases

- If the two lines M and $\vec{M}$ intersect at the point A and $\overline{BB'} \parallel \overline{CC'}$
 , then $\frac{AB}{AC} = \frac{A'B'}{A'C'}$
 and conversely if $\frac{AB}{AC} = \frac{A'B'}{A'C'}$, then $\overline{BB'} \parallel \overline{CC'}$



2 Talis' special theorem :

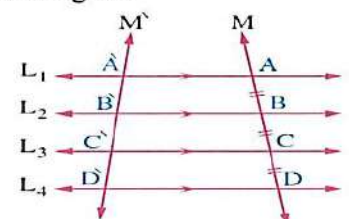
If the lengths of the segments on the transversal are equal , then the lengths of the segments on any other transversal will be also equal.

In the opposite figure :

If $L_1 \parallel L_2 \parallel L_3 \parallel L_4$,

M and $\vec{M}$ are two transversals to them

and if $AB = BC = CD$, then $A'B' = B'C' = C'D'$



SIMPLEST MATHS BOOKLETS

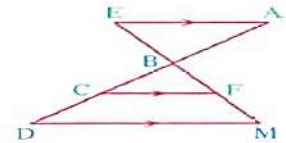
Ex.1) Choose the correct answer :

(1) In the opposite figure :

$AB : BC : CD = \dots\dots\dots$

- (a) $AE : FC : MD$
(c) $EB : BC : CD$

- (b) $EB : BF : FM$
(d) $EB : EF : EM$

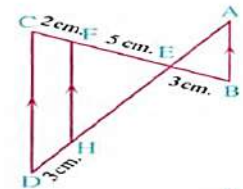


(2) In the opposite figure :

$AH = \dots\dots\dots$ cm.

- (a) 6
(c) 10

- (b) 7.5
(d) 12

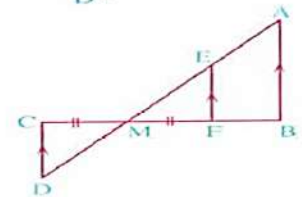


(3) In the opposite figure :

If $DA = 21$ cm. , $MC = 5$ cm. , $FB = 4$ cm.
 , then $AE = \dots\dots\dots$ cm.

- (a) 3
(c) 6

- (b) 5
(d) 4

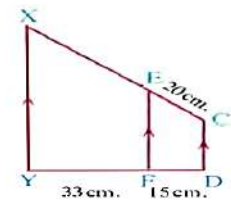


(4) In the opposite figure :

$\overline{CD} \parallel \overline{EF} \parallel \overline{XY}$, $CE = 20$ cm.
 , $DF = 15$ cm. , $FY = 33$ cm.
 , then the length of $\overline{CX} = \dots\dots\dots$ cm.

- (a) 48 (b) 64 (c) 44

- (d) 21

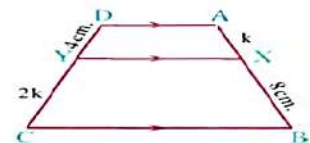


(5) In the opposite figure :

If $\overline{AD} \parallel \overline{XY} \parallel \overline{BC}$, then
 $AX = \dots\dots\dots$ cm.

- (a) $\frac{3}{8}$ (b) 4 (c) 16

- (d) 32

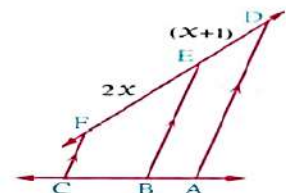


(6) In the opposite figure :

If $\overline{AD} \parallel \overline{BE} \parallel \overline{CF}$, $AB = 3$ cm.
 , $BC = 5$ cm. , $DE = (x + 1)$ cm.
 , $EF = 2x$ cm. , then $x = \dots\dots\dots$ cm.

- (a) 3 (b) 4 (c) 5

- (d) 8

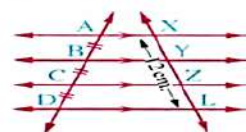


(7) In the opposite figure :

If $AB = BC = CD$,
 $XL = 12$ cm. , then $XZ = \dots\dots\dots$

- (a) 4 cm. (b) YL (c) AC

- (d) BC

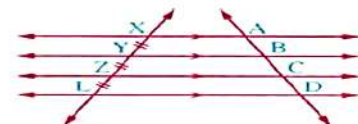


(8) In the opposite figure :

If $BD = 14$ cm.
 , $AC = \dots\dots\dots$ cm.

- (a) 7 (b) 14 (c) 21

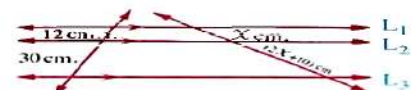
- (d) 28



(9) In the opposite figure :

$x = \dots\dots\dots$ cm.

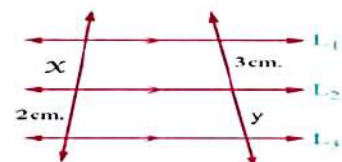
- (a) 10 (b) 20
(c) 15 (d) 8



(10) In the opposite figure :

If $x > 2$, then $\dots\dots\dots$

- (a) $y = 3$ (b) $y > 3$
(c) $y < 3$ (d) $y \leq 3$



SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

1

Write what each of the following ratios equals using the opposite figure :

$$(1) \frac{AB}{BC} = \frac{DE}{\dots\dots\dots}$$

$$(2) \frac{AC}{BC} = \frac{\dots\dots\dots}{EF}$$

$$(3) \frac{MA}{AB} = \frac{MD}{\dots\dots\dots}$$

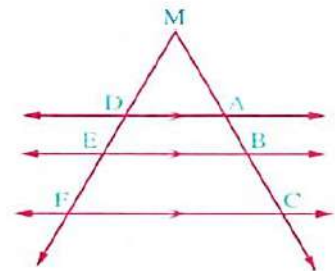
$$(4) \frac{AC}{AB} = \frac{\dots\dots\dots}{DE}$$

$$(5) \frac{MB}{AB} = \frac{\dots\dots\dots}{DE}$$

$$(6) \frac{MC}{AC} = \frac{MF}{\dots\dots\dots}$$

$$(7) \frac{BC}{MB} = \frac{EF}{\dots\dots\dots}$$

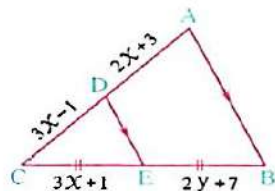
$$(8) \frac{DF}{MF} = \frac{AC}{\dots\dots\dots}$$



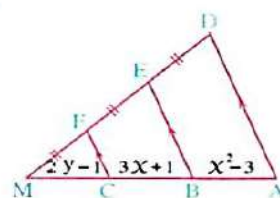
2

In each of the following figures , calculate the numerical values of x and y (Lengths are measured in centimetres) :

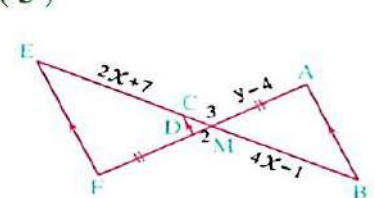
(1)



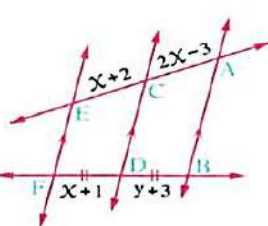
(2)



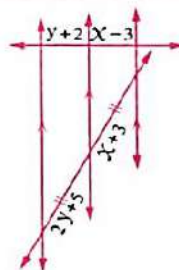
(3)



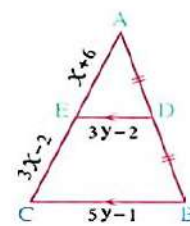
(4)



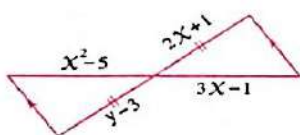
(5)



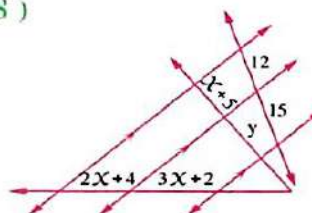
(6)



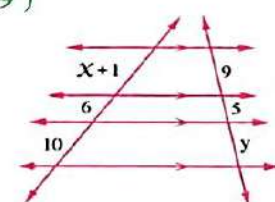
(7)



(8)



(9)



3

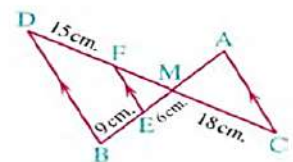
In the opposite figure :

$$\overline{AB} \cap \overline{CD} = \{M\}, E \in \overline{MB},$$

$$F \in \overline{MD} \text{ and } \overline{AC} \parallel \overline{FE} \parallel \overline{DB}$$

Find : (1) The length of $\overline{MF}$

(2) The length of $\overline{AM}$



« 10 cm. , 10.8 cm. »

4

$$\overline{AB} \cap \overline{CD} = \{E\}, X \in \overline{AB}, Y \in \overline{CD}, \text{ and } \overline{XY} \parallel \overline{BD} \parallel \overline{AC}$$

Prove that : $AX \times ED = CY \times EB$

SIMPLEST MATHS BOOKLETS

5 ABCD is a quadrilateral in which $\overline{AB} \parallel \overline{CD}$, its diagonals intersect at M and E is the midpoint of $\overline{BC}$, $\overline{EF} \parallel \overline{BA}$ and intersects $\overline{BD}$ at X, $\overline{AC}$ at Y and $\overline{AD}$ at F

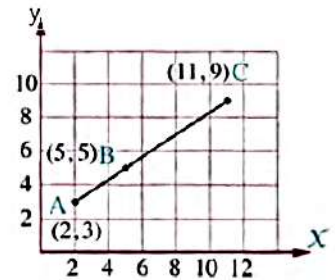
Prove that : (1) $EY = \frac{1}{2} AB$

(2) $\frac{AY}{CM} = \frac{BX}{DM}$

6 Logical thinking :

From the figure, find the value of $\frac{AB}{BC}$ in different methods, if possible.

Did you get the same result ?



Lesson

3

Angle bisector and proportional parts

Theorem 3

The bisector of the interior or exterior angle of a triangle at any vertex divides the opposite base of the triangle internally or externally into two parts, the ratio of their lengths is equal to the ratio of the lengths of the other two sides of the triangle.

Important Remarks

i.e. The interior and exterior bisectors for any angle in the triangle are perpendicular

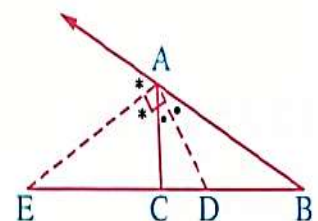
1 In the opposite figure :

If $\overrightarrow{AD}$, $\overrightarrow{AE}$ are the bisectors of the angle A and the exterior angle of $\triangle ABC$ at A respectively

, then $\frac{BD}{DC} = \frac{AB}{AC}$, $\frac{BE}{EC} = \frac{AB}{AC} \therefore \frac{BD}{DC} = \frac{BE}{EC}$

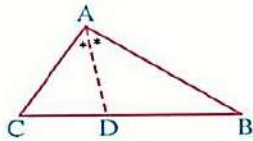
$\therefore$ The base $\overline{BC}$ is divided internally at D, externally at E by the same ratio (AB : AC) and we notice that : the two bisectors $\overrightarrow{AD}$ and $\overrightarrow{AE}$ are perpendicular.

i.e. $m(\angle DAE) = 90^\circ$

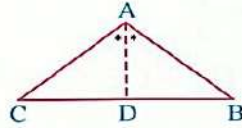


SIMPLEST MATHS BOOKLETS

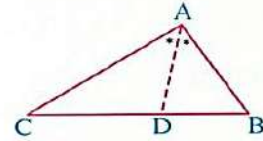
2 If $\overrightarrow{AD}$ bisects $\angle BAC$ and intersects $\overline{BC}$ at D, then D takes one of the following :



If $AB > AC$
 , then $BD > DC$
i.e. D is nearer to C than to B



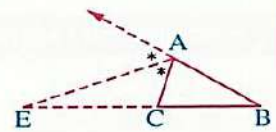
If $AB = AC$
 , then $BD = DC$
i.e. D is equidistant from each of B and C



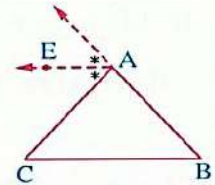
If $AB < AC$
 , then $BD < DC$
i.e. D is nearer to B than to C

3 If $\overrightarrow{AE}$ bisects the exterior angle of $\triangle ABC$ at A, where $E \notin \overline{BC}$, then E takes one of the following cases :

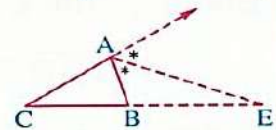
① If $AB > AC$, then $BE > EC$ *i.e.* $E \in \overline{BC}$



② If $AB = AC$, then $\overrightarrow{AE} \parallel \overline{BC}$
i.e. The exterior bisector of the vertex of isosceles triangle is paralleling to the base.



③ If $AB < AC$, then $BE < EC$ *i.e.* $E \in \overline{CB}$



Finding the lengths of the interior and the exterior bisectors of an angle of a triangle

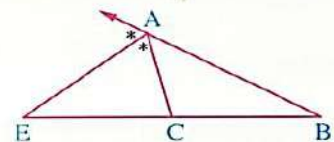
Well known problem

If $\overrightarrow{AD}$ bisects $\angle A$ in $\triangle ABC$ internally and intersects $\overline{BC}$ at D
 , then $AD = \sqrt{AB \times AC - BD \times DC}$

Remark

In the opposite figure :

If $\overrightarrow{AE}$ bisects $\angle BAC$ externally and intersects $\overline{BC}$ at E
 , then $AE = \sqrt{BE \times EC - AB \times AC}$



SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

(1) In the opposite figure :

CD = cm.

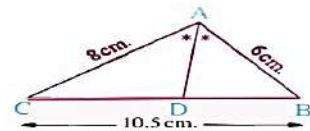
- (a) 4.5 (b) 5 (c) 4.9 (d) 6



(2) In the opposite figure :

BD = cm.

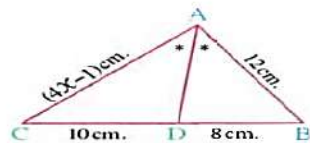
- (a) 4 (b) $\frac{2}{3}$ (c) 4.5 (d) 45



(3) In the opposite figure :

X =

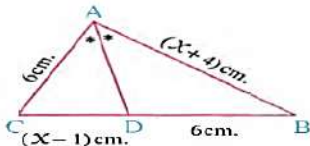
- (a) 4 (b) 3 (c) 4.5 (d) 6



(4) In the opposite figure :

X = cm.

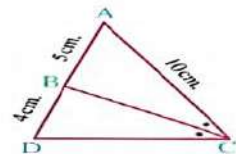
- (a) 6 (b) 5
(c) 8 (d) 10



(5) In the opposite figure :

CB = cm.

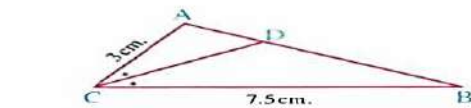
- (a) 8 (b) $4\sqrt{2}$
(c) $2\sqrt{15}$ (d) 6



(6) In the opposite figure :

$\overline{CD}$ bisects $\angle C$,
AC = 3 cm. , BC = 7.5 cm.
, then AD : BD =

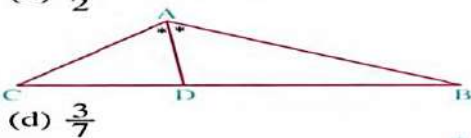
- (a) $\frac{3}{5}$ (b) $\frac{2}{3}$ (c) $\frac{2}{5}$



(7) In the opposite figure :

If AB : AC : BC = 5 : 3 : 7 , then BD : DC =

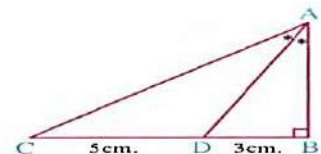
- (a) $\frac{5}{3}$ (b) $\frac{5}{7}$ (c) $\frac{3}{5}$



(8) In the opposite figure :

AB = cm.

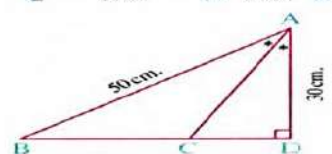
- (a) 4 (b) 5 (c) 6 (d) 7



(9) In the opposite figure :

The perimeter of $\triangle ABC \approx$ cm.

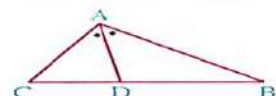
- (a) 123.5 (b) 375
(c) 98.5 (d) 108.5



(10) In the opposite figure :

$\overline{AD}$ bisects $\angle A$, then $AB \times CD =$

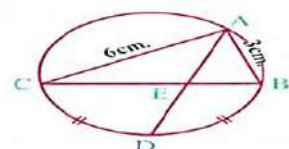
- (a) $AC \times BD$ (b) $(AD)^2$
(c) $AD \times BD$ (d) $AC \times AB$



(11) In the opposite figure :

$\frac{BE}{BC} =$

- (a) $\frac{1}{2}$ (b) 2
(c) $\frac{1}{3}$ (d) 3

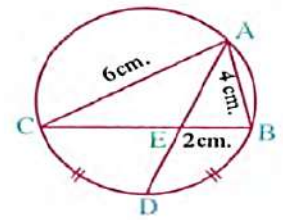


SIMPLEST MATHS BOOKLETS

(12) In the opposite figure :

The length of $\overline{DE}$ = cm.

- (a) 4 (b) 2
(c) $\sqrt{2}$ (d) $3\sqrt{2}$



(13) The exterior bisector of the vertex angle of an isosceles triangle the base.

- (a) bisects (b) perpendicular to
(c) intersect (d) parallel

(14) The bisector of the exterior angle of an equilateral triangle the side opposite to the vertex of this angle.

- (a) bisects (b) congruent to (c) parallel (d) perpendicular to

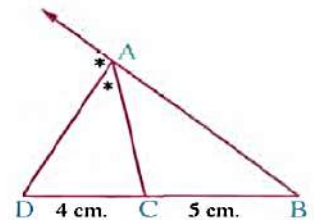
(15) The measure of the angle included between the interior and the exterior bisector at any vertex of angles of the triangle equal

- (a) 45° (b) 90° (c) 135° (d) 180°

(16) In the opposite figure :

$AB : AC = \dots\dots\dots$

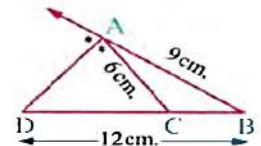
- (a) 5 : 4 (b) 5 : 9
(c) 9 : 5 (d) 9 : 4



(17) In the opposite figure :

$CD = \dots\dots\dots$ cm.

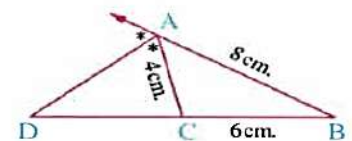
- (a) 8 (b) 6
(c) 4.8 (d) 5



(18) In the opposite figure :

$CD = \dots\dots\dots$ cm.

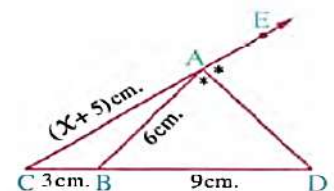
- (a) 2 (b) 6 (c) 4 (d) 8



(19) In the opposite figure :

$\overrightarrow{AD}$ bisects $\angle BAE$, if $AC = (x + 5)$ cm. ,
 $AB = 6$ cm. , $BC = 3$ cm. , $BD = 9$ cm.
, then $x = \dots\dots\dots$ cm.

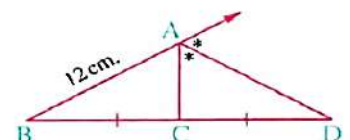
- (a) 4 (b) 3 (c) 2 (d) 6



(20) In the opposite figure :

$AC = \dots\dots\dots$ cm.

- (a) 3 (b) 4 (c) 6 (d) 8



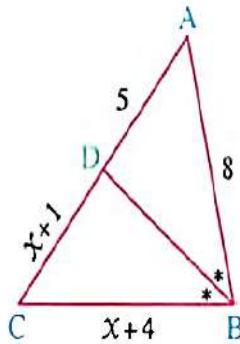
SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

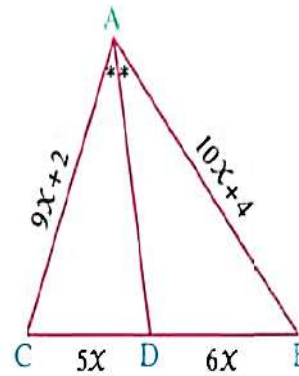
1

In each of the following figures , find the value of x (Lengths are measured in centimetres) :

(1)



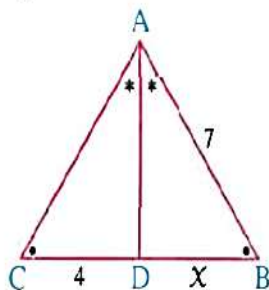
(2)



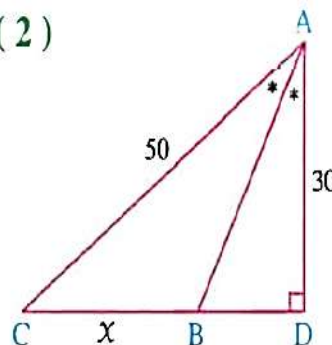
2

In each of the following figures , find the value of x (Lengths are measured in centimetres) , then find the perimeter of $\triangle ABC$:

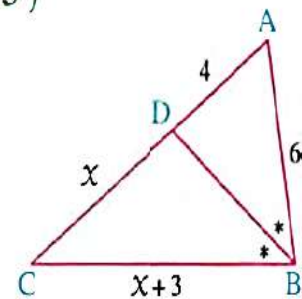
(1)



(2)



(3)



3

ABC is a triangle in which : $AB = 8$ cm. , $AC = 6$ cm. , $BC = 7$ cm. , $\overline{AD}$ bisects $\angle BAC$ and intersects $\overline{BC}$ at D Find the length of each of : $\overline{DB}$, $\overline{DC}$ « 4 cm. , 3 cm. »

4

ABC is a triangle , its perimeter is 27 cm. , $\overline{BD}$ bisects $\angle B$ and intersects $\overline{AC}$ at D If $AD = 4$ cm. and $CD = 5$ cm. , find the length of each of : $\overline{AB}$, $\overline{BC}$ and $\overline{BD}$ « 8 cm. , 10 cm. , $2\sqrt{15}$ cm. »

SIMPLEST MATHS BOOKLETS

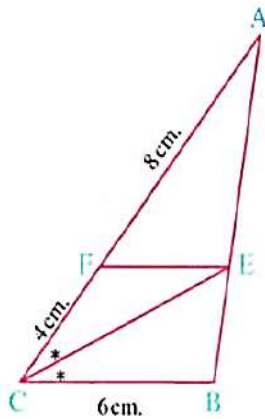
5  ABC is a right-angled triangle at B , draw $\overrightarrow{AD}$ bisects $\angle A$, and intersects $\overline{BC}$ at D
If the length of $\overline{BD}$ equals 24 cm. , $BA : AC = 3 : 5$, find the perimeter of ΔABC « 192 cm. »

6  ABC is a triangle in which $AB = 8$ cm. , $AC = 4$ cm. and $BC = 6$ cm. , $\overrightarrow{AD}$ bisects $\angle A$ and intersects $\overline{BC}$ at D , $\overrightarrow{AE}$ bisects the exterior angle at A and intersects $\overline{BC}$ at E
Find the length of each of : $\overline{DE}$, $\overline{AD}$ and $\overline{AE}$ « 8 cm. , $2\sqrt{6}$ cm. , $2\sqrt{10}$ cm. »

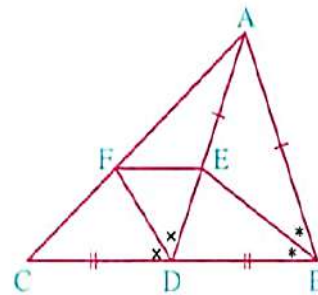
7  ABC is a triangle in which $AB = 3$ cm. , $BC = 7$ cm. , $CA = 6$ cm. , $\overrightarrow{AD}$ bisects $\angle A$ and intersects $\overline{BC}$ at D , $\overrightarrow{AE}$ bisects the exterior angle of the triangle at A and intersects $\overline{CB}$ at E
(1) **Prove that :** $\overline{AB}$ is a median in the triangle ACE
(2) **Find the ratio of :** The area of ΔADE to the area of ΔACE « $\frac{2}{3}$ »

8  In each of the following two figures , prove that $\overline{EF} \parallel \overline{BC}$:

(1)



(2)



SIMPLEST MATHS BOOKLETS

Lesson

4

Follow : Angle bisector and proportional parts (Converse of theorem 3)

Converse of theorem 3

In the opposite two figures :

- If $D \in \overline{BC}$ (Fig. 1)
such that : $\frac{BD}{DC} = \frac{BA}{AC}$
 , then $\overline{AD}$ bisects $\angle BAC$
- If $D \in \overline{BC}$, $D \notin \overline{BC}$ (Fig. 2)
such that : $\frac{BD}{DC} = \frac{BA}{AC}$
 , then $\overline{AD}$ bisects the exterior angle of $\triangle ABC$ at A

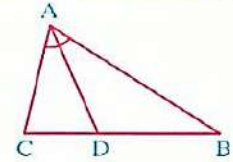


Fig. (1)

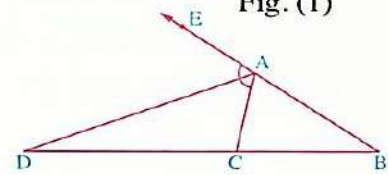


Fig. (2)

Example 1

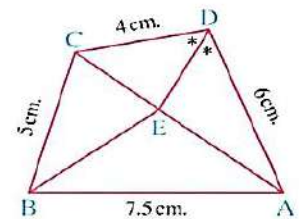
In the opposite figure :

ABCD is a quadrilateral in which $AB = 7.5$ cm.

, $BC = 5$ cm. , $CD = 4$ cm. , $AD = 6$ cm.

, $\overline{DE}$ bisects $\angle ADC$ and intersects $\overline{AC}$ at E

Prove that : $\overline{BE}$ bisects $\angle ABC$



Solution

In $\triangle ACD$: $\therefore \overline{DE}$ bisects $\angle ADC$

$$\therefore \frac{AE}{EC} = \frac{AD}{DC} = \frac{6}{4} = \frac{3}{2}$$

$$\therefore \frac{AB}{BC} = \frac{7.5}{5} = \frac{3}{2}$$

$$\therefore \frac{AE}{EC} = \frac{AB}{BC}$$

$\therefore$ In $\triangle ABC$: $\overline{BE}$ bisects $\angle ABC$

(Q.E.D.)

Fact

The bisectors of angles of a triangle are concurrent.

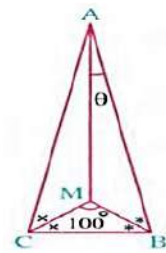
SIMPLEST MATHS BOOKLETS

Ex.1) Choose the correct answer :

(1) In the opposite figure :

$$\theta = \dots\dots\dots$$

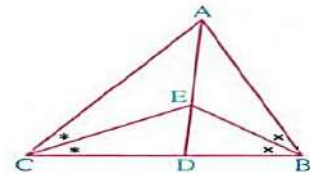
- (a) 10° (b) 20° (c) 40° (d) 80°



(2) In the opposite figure :

If $\overline{BE}$ bisects $\angle ABD$, $\overline{CE}$ bisects $\angle ACD$, then

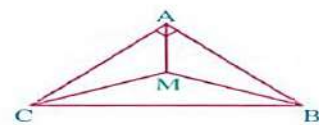
- (a) D is a midpoint of $\overline{BC}$
 (b) E is the midpoint of $\overline{AD}$
 (c) E divides $\overline{AD}$ by the ratio 2 : 1 from the direction of point A
 (d) $\overline{AD}$ bisects $\angle BAC$



(3) In the opposite figure :

$\overline{AB} \perp \overline{AC}$, M is the point of intersection of the bisectors of the interior angles of $\triangle ABC$, then $m(\angle BMC) = \dots\dots\dots$

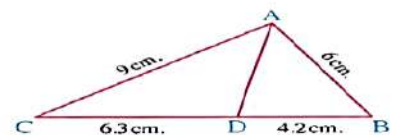
- (a) 100° (b) 120° (c) 135° (d) 145°



(4) In the opposite figure :

which of the following statements is true ?

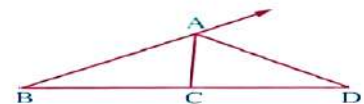
- (a) $\triangle BAD \sim \triangle BCA$
 (b) $AB \times AC = BD \times DC$
 (c) $m(\angle BAD) = m(\angle CAD)$
 (d) $AD = \sqrt{BD \times DC - AB \times AC}$



(5) In the opposite figure :

Which of the following conditions is sufficient to prove that $\overline{AD}$ bisects the exterior angle at the vertex A ?

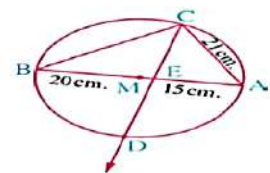
- (a) $\frac{AD}{AC} = \frac{DB}{BC}$ (b) $\frac{AB}{AC} = \frac{BD}{BC}$
 (c) $\frac{AB}{AC} = \frac{CD}{BD}$ (d) $AB \times DC = AC \times DB$



(6) In the opposite figure :

Circle M in which , $\overline{AB}$ is a diameter , $E \in \overline{AB}$, if $AE = 15$ cm. , $BE = 20$ cm. , $AC = 21$ cm. , $\overline{CE}$ intersect circle M at D , then $m(\widehat{AD}) = \dots\dots\dots^\circ$

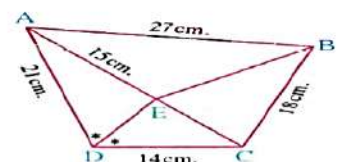
- (a) 45 (b) 90
 (c) 22.5 (d) 60



(7) In the opposite figure :

which of the following statements is false ?

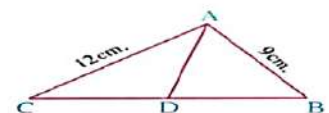
- (a) $CE = 10$ cm. (b) $\overline{BE}$ bisects $\angle ABC$
 (c) $BE = 4\sqrt{21}$ cm. (d) $DE = 12\sqrt{2}$ cm.



(8) In the opposite figure :

If a $(\triangle ABD) = 30$ cm² , a $(\triangle ACD) = 40$ cm² , then $\overline{AD}$ is

- (a) perpendicular to $\overline{BC}$ (b) bisects $\angle BAC$
 (c) passes through the midpoint of $\overline{BC}$ (d) All the previous



SIMPLEST MATHS BOOKLETS

Ex.2) Answer the following:

1

ABC is a triangle in which : $AB = 6 \text{ cm.}$, $AC = 9 \text{ cm.}$, $BC = 10.5 \text{ cm.}$, $D \in \overline{BC}$, where $BD = 4.2 \text{ cm.}$ **Prove that :** $\overrightarrow{AD}$ bisects $\angle BAC$

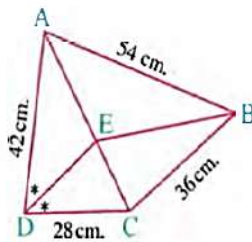
2

ABC is a triangle in which $AB = 6 \text{ cm.}$, $BC = 4 \text{ cm.}$, $CA = 3.6 \text{ cm.}$, $D \in \overline{BC}$ such that $CD = 6 \text{ cm.}$ **Prove that :** $\overrightarrow{AD}$ bisects the exterior angle of $\triangle ABC$ at A

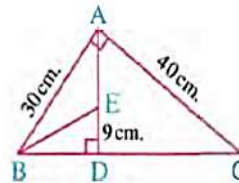
3

 In each of the following figures , prove that : $\overrightarrow{BE}$ bisects $\angle ABC$

(1)



(2)



4

ABCD is a quadrilateral in which $AB = 6 \text{ cm.}$, $BC = 9 \text{ cm.}$, $CD = 6 \text{ cm.}$, $AD = 4 \text{ cm.}$, $\overrightarrow{AE}$ bisects $\angle A$ and intersects $\overline{BD}$ at E

(1) Find the value of the ratio : $\frac{BE}{ED}$

(2) Prove that : $\overrightarrow{CE}$ bisects $\angle BCD$

« $\frac{3}{2}$ »

5

 ABCD is a quadrilateral in which $AB = 18 \text{ cm.}$, $BC = 12 \text{ cm.}$, $E \in \overline{AD}$, where $2AE = 3ED$, draw $\overrightarrow{EF} \parallel \overline{DC}$ and intersects $\overline{AC}$ at F

Prove that : $\overrightarrow{BF}$ bisects $\angle ABC$

6

 ABC is a triangle , $D \in \overline{BC}$, $D \notin \overline{BC}$, where $CD = AB$, draw $\overrightarrow{CE} \parallel \overline{DA}$ and intersects $\overline{AB}$ at E , draw $\overrightarrow{EF} \parallel \overline{BC}$ and intersects $\overline{AC}$ at F

Prove that : $\overrightarrow{BF}$ bisects $\angle ABC$

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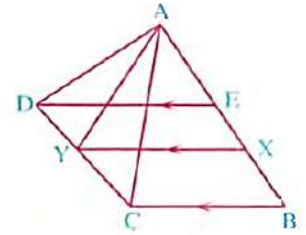
7

 In the opposite figure :

$$\overline{ED} \parallel \overline{XY} \parallel \overline{BC}$$

$$\text{and } AD \times BX = AC \times EX$$

Prove that : $\overrightarrow{AY}$ bisects $\angle CAD$



8

 Two circles M and N are touching externally at A , a straight line is drawn parallel to $\overline{MN}$ and intersects the circle M at B , C and the circle N at D , E respectively.

If $\overline{BM} \cap \overline{EN} = \{F\}$, **prove that :** $\overrightarrow{FA}$ bisects $\angle MFN$

9

 $\overline{AB}$ is a diameter of a circle , $\overline{AC}$ is a chord in it , $\overline{CD}$ is a tangent drawn to the circle at C and intersects $\overline{AB}$ at D. If $E \in \overline{AB}$, where $\frac{DB}{BE} = \frac{DC}{CE}$

Prove that : (1) $\overrightarrow{CA}$ bisects the exterior angle of ΔCDE at C

$$(2) \frac{DA}{DB} = \frac{AE}{BE}$$

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Answers of pre-requirements

First Multiple choice questions

- (1) d (2) b (3) c (4) d
(5) d (6) c (7) a

Second Essay questions

1

(1) $\therefore a = 1, b = -6, c = 1$

$$\therefore x = \frac{6 \pm \sqrt{36 - 4 \times 1 \times 1}}{2 \times 1} = \frac{6 \pm 4\sqrt{2}}{2} = 3 \pm 2\sqrt{2}$$

$$\therefore S.S. = \{5.8, 0.2\}$$

(2) $\therefore a = 1, b = 3, c = 5$

$$\therefore x = \frac{-3 \pm \sqrt{9 - 4 \times 1 \times 5}}{2 \times 1} = \frac{-3 \pm \sqrt{-11}}{2}$$

$$\therefore S.S. = \emptyset$$

(3) $\therefore a = 2, b = 3, c = -4$

$$\therefore x = \frac{-3 \pm \sqrt{9 - 4 \times 2 \times -4}}{2 \times 2} = \frac{-3 \pm \sqrt{41}}{4}$$

$$\therefore S.S. = \{0.9, -2.4\}$$

(4) $\therefore a = 3, b = 0, c = -65$

$$\therefore x = \frac{\pm \sqrt{-4 \times 3 \times -65}}{2 \times 3} = \frac{\pm \sqrt{780}}{6}$$

$$\therefore S.S. = \{4.7, -4.7\}$$

2

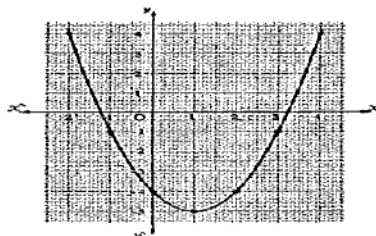
(1) $\therefore a = 1, b = -2, c = -4$

$$\therefore x = \frac{2 \pm \sqrt{4 - 4 \times 1 \times -4}}{2 \times 1} = \frac{2 \pm \sqrt{20}}{2} = \frac{2 \pm 2\sqrt{5}}{2} = 1 \pm \sqrt{5}$$

$$\therefore S.S. = \{-1.2, 3.2\}$$

Let $f(x) = x^2 - 2x - 4$

x	-2	-1	0	1	2	3	4
y	4	-1	-4	-5	-4	-1	4



From the graph :

$$S.S. = \{-1.2, 3.2\} \text{ approximately}$$

(2) $\therefore a = -1, b = 3, c = 2$

$$\therefore x = \frac{-3 \pm \sqrt{9 - 4 \times -1 \times 2}}{2 \times -1} = \frac{-3 \pm \sqrt{17}}{-2}$$

$$\therefore S.S. = \{-0.6, 3.6\}$$

Let $f(x) = -x^2 + 3x + 2$

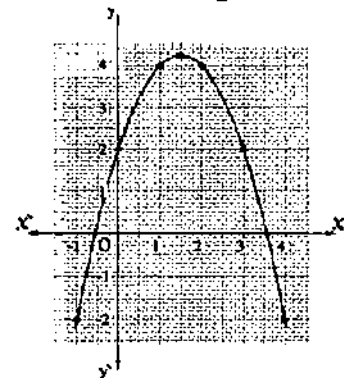
x	-1	0	1	2	3	4
y	-2	2	4	4	2	-2

$\therefore$ The X-coordinate of the curve vertex point

$$= \frac{-b}{2a} = \frac{-3}{2 \times -1} = \frac{3}{2} = 1\frac{1}{2}$$

$$f\left(\frac{-b}{2a}\right) = f\left(1\frac{1}{2}\right) = -\left(1\frac{1}{2}\right)^2 + 3 \times 1\frac{1}{2} + 2 = 4\frac{1}{4}$$

$\therefore$ The vertex point is $\left(1\frac{1}{2}, 4\frac{1}{4}\right)$



From the graph :

$$S.S. = \{-0.6, 3.6\} \text{ approximately}$$

3

(1) $\therefore 78 = \frac{n}{2}(1+n)$

$$\therefore \frac{n^2}{2} + \frac{n}{2} - 78 = 0 \quad (\text{multiplying by 2})$$

$$\therefore n^2 + n - 156 = 0 \quad \therefore (n-12)(n+13) = 0$$

$$\therefore n = 12 \text{ or } n = -13 \text{ (refused)}$$

$$\therefore \text{no. of integers} = 12 \text{ integers.}$$

(2) $\therefore 171 = \frac{n}{2}(1+n)$

$$\therefore \frac{n^2}{2} + \frac{n}{2} - 171 = 0 \quad (\text{multiplying by 2})$$

$$\therefore n^2 + n - 342 = 0 \quad \therefore (n-18)(n+19) = 0$$

$$\therefore n = 18 \text{ or } n = -19 \text{ (refused)}$$

$$\therefore \text{no. of integers} = 18 \text{ integers.}$$

(3) $\therefore 253 = \frac{n}{2}(1+n)$

$$\therefore \frac{n^2}{2} + \frac{n}{2} - 253 = 0 \quad (\text{multiplying by 2})$$

$$\therefore n^2 + n - 506 = 0$$

$$\therefore (n-22)(n+23) = 0$$

$$\therefore n = 22 \text{ or } n = -23 \text{ (refused)}$$

$$\therefore \text{no. of integers} = 22 \text{ integers.}$$

(4) $\therefore 465 = \frac{n}{2}(1+n)$

$$\therefore \frac{n^2}{2} + \frac{n}{2} - 465 = 0 \quad (\text{multiplying by 2})$$

$$\therefore n^2 + n - 930 = 0 \quad \therefore (n-30)(n+31) = 0$$

$$\therefore n = 30 \text{ or } n = -31 \text{ (refused)}$$

$$\therefore \text{no. of integers} = 30 \text{ integers.}$$

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Answers of exercise 1

First Multiple choice questions

- (1) c (2) d (3) a (4) b (5) c
 (6) c (7) b (8) b (9) c (10) a
 (11) b (12) b (13) b (14) a (15) c
 (16) d (17) c (18) c (19) c (20) d
 (21) a (22) b (23) d

Second Essay questions

1

- (1) $6 + i - 12i^2 = 18 + i$
 (2) $4 - 20i + 25i^2 = -21 - 20i$
 (3) $9 - 12i + 4i^2 + 3 + 2i = 8 - 10i$
 (4) $[(1+i)^2]^2 = [1+2i+i^2]^2 = (2i)^2 = 4i^2 = -4$
 (5) $[(1+i)^2]^2 - [(1-i)^2]^2$
 $= (1+2i+i^2)^2 - (1-2i+i^2)^2$
 $= (2i)^2 - (-2i)^2 = 4i^2 - 4i^2 = \text{zero}$
 (6) $[(1-i)^2]^5 = (1-2i+i^2)^5 = (-2i)^5$
 $= -32i^5 = -32i$

2

- (1) $\frac{4-5i}{7i} \times \frac{-7i}{-7i} = \frac{-28i+35i^2}{-49i^2} = \frac{5}{7} - \frac{4}{7}i$
 (2) $\frac{26}{3-2i} \times \frac{3+2i}{3+2i} = \frac{78+52i}{9-4i^2} = \frac{78+52i}{13} = 6+4i$
 (3) $\frac{2-3i}{3+i} \times \frac{3-i}{3-i} = \frac{6-11i+3i^2}{9-i^2} = \frac{3}{10} - \frac{11}{10}i$
 (4) $\frac{3+4i}{5-2i} \times \frac{5+2i}{5+2i} = \frac{15+26i+8i^2}{25-4i^2} = \frac{7}{29} + \frac{26}{29}i$
 (5) $\frac{(3+2i)(2-i)}{3+i} = \frac{6+i-2i^2}{3+i} = \frac{8+i}{3+i}$
 $\therefore \frac{8+i}{3+i} \times \frac{3-i}{3-i} = \frac{24-5i-i^2}{9-i^2} = \frac{25-5i}{10} = \frac{5}{2} - \frac{1}{2}i$
 (6) $\frac{(3+i)(3-i)}{3-4i} = \frac{9-i^2}{3-4i} = \frac{10}{3-4i}$
 $\therefore \frac{10}{3-4i} \times \frac{3+4i}{3+4i} = \frac{10(3+4i)}{9-16i^2} = \frac{10(3+4i)}{25}$
 $= \frac{6}{5} + \frac{8}{5}i$

3

- (1) $\therefore 3x^2 + 12 = 0 \quad \therefore 3x^2 = -12$
 $\therefore x^2 = -4 \quad \therefore x = \pm\sqrt{-4}$
 $\therefore x = \pm\sqrt{4i^2} \quad \therefore x = \pm 2i$
 (2) $\therefore 4x^2 + 100 = 75 \quad \therefore 4x^2 = -25$
 $\therefore x^2 = -\frac{25}{4} \quad \therefore x = \pm\sqrt{-\frac{25}{4}}$
 $\therefore x = \pm\sqrt{\frac{25}{4}i^2} \quad \therefore x = \pm\frac{5}{2}i$
 (3) $x = \frac{4 \pm \sqrt{(-4)^2 - 4 \times 1 \times 5}}{2} = \frac{4 \pm \sqrt{-4}}{2}$
 $= \frac{4 \pm \sqrt{4}i}{2} = 2 \pm i$
 (4) $x = \frac{-6 \pm \sqrt{(6)^2 - 4 \times 2 \times 5}}{2 \times 2} = \frac{-6 \pm \sqrt{-4}}{4}$
 $= \frac{-6 \pm \sqrt{4}i}{4}$
 $= \frac{-3}{2} \pm \frac{1}{2}i$

4

- (1) $\therefore (2x-3) + (3y+1)i = 7 + 10i$
 $\therefore 2x-3 = 7 \quad \therefore 2x = 10$
 $\therefore x = 5, 3y+1 = 10$
 $\therefore 3y = 9 \quad \therefore y = 3$
 (2) $\therefore (2x-y) + (x-2y)i = 5 + i$
 $\therefore 2x-y = 5 \quad (1), x-2y = 1$
 Multiply (1) by -2 : $\therefore -4x + 2y = -10$
 adding (2) and (3): $\therefore -3x = -9$
 $\therefore x = 3 \quad \therefore y = 1$
 (3) $\therefore 3x + xi - 2y + yi = 5$
 $\therefore (3x-2y) + (x+y)i = 5$
 $\therefore 3x-2y = 5 \quad (1), x+y = 0 \quad (2)$
 Multiply (2) by 2: $\therefore 2x + 2y = 0$ (3)
 adding (1) and (3): $\therefore 5x = 5$
 $\therefore x = 1 \quad \therefore y = -1$

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$$\begin{aligned}
 (4) \text{ L.H.S.} &= \frac{(2+i)(2-i)}{3+4i} = \frac{4-i^2}{3+4i} = \frac{5}{3+4i} \\
 &= \frac{5}{3+4i} \times \frac{3-4i}{3-4i} = \frac{5(3-4i)}{9-16i^2} \\
 &= \frac{5(3-4i)}{25} = \frac{3}{5} - \frac{4}{5}i \\
 \therefore \frac{3}{5} - \frac{4}{5}i &= x + yi \quad \therefore x = \frac{3}{5}, y = -\frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 (5) \quad \therefore x &= \frac{13}{5-i} \times \frac{5+i}{5+i} = \frac{13(5+i)}{25-i^2} = \frac{13(5+i)}{26} = \frac{5}{2} + \frac{i}{2} \\
 y &= \frac{3+2i}{1+i} \times \frac{1-i}{1-i} = \frac{3-i-2i^2}{1-i^2} = \frac{5-i}{2} = \frac{5}{2} - \frac{i}{2} \\
 \therefore x, y &\text{ are two conjugate numbers.}
 \end{aligned}$$

$$\begin{aligned}
 (6) \quad \text{R.H.S.} &= \frac{2+i}{2-i} \times \frac{2+i}{2+i} = \frac{4+4i+i^2}{4-i^2} \\
 &= \frac{3+4i}{5} = \frac{3}{5} + \frac{4}{5}i \\
 \therefore a + bi &= \frac{3}{5} + \frac{4}{5}i \quad \therefore a = \frac{3}{5}, b = \frac{4}{5} \\
 \therefore a^2 + b^2 &= \left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = 1
 \end{aligned}$$

Higher skills

(1) c (2) a (3) d (4) d (5) d

Instructions to solve (1) :

(1) $\therefore L$ is a root of the equation : $x^2 + 1 = 0$

$\therefore L^2 + 1 = 0 \quad \therefore L^2 = -1$

$\therefore L^{2018} = (L^2)^{1009} = (-1)^{1009} = -1$

Similarly $M^{2018} = -1$

$\therefore L^{2018} + M^{2018} = (-1) + (-1) = -2$

(2) $(1+i)^{2020} = [(1+i)^2]^{1010} = (2i)^{1010}$

$\therefore (1-i)^{2020} = [(1-i)^2]^{1010} = (-2i)^{1010}$
 $= (2i)^{1010}$

$\therefore (1+i)^{2020} = (1-i)^{2020}$

(3) $\left(\frac{1-i}{1+i}\right)^{100} = \left(\frac{(1-i)^2}{(1+i)^2}\right)^{50} = \left(\frac{-2i}{2i}\right)^{50} = 1$

(4) $(2+i)^{-1} = \frac{1}{2+i} \times \frac{2-i}{2-i} = \frac{2-i}{4+1} = \frac{2-i}{5}$

$\therefore$ Conjugate of the number $(2+i)^{-1}$ is $\frac{2+i}{5}$

(5) $x^2 + 4 = x^2 - 4i^2 = (x-2i)(x+2i)$

2

$$\begin{aligned}
 \therefore 7i &= (x+3i)(y-i) - 9 \\
 &= xy - xi + 3yi - 3i^2 - 9 \\
 &= xy - xi + 3yi - 6 \\
 &= (xy-6) + (-x+3y)i
 \end{aligned}$$

$\therefore xy-6=0 \quad \therefore xy=6$

$\therefore -x+3y=7 \quad \therefore x=3y-7$

$\therefore y(3y-7)=6 \quad \therefore 3y^2-7y=6$

$\therefore 3y^2-7y-6=0 \quad \therefore (y-3)(3y+2)=0$

$\therefore y=3 \quad \therefore x=2$

or $y = -\frac{2}{3} \quad \therefore x = -9$

3

$$x = \frac{2+i}{2-i} \times \frac{2+i}{2+i} = \frac{4+4i+i^2}{4-i^2} = \frac{3+4i}{5} = \frac{3}{5} + \frac{4}{5}i$$

$$y = \frac{2+3i}{2+i} \times \frac{2-i}{2-i} = \frac{4+4i-3i^2}{4-i^2} = \frac{7+4i}{5} = \frac{7}{5} + \frac{4}{5}i$$

$\therefore 2x-y = a+bi$

$\therefore \frac{6}{5} + \frac{8}{5}i - \frac{7}{5} - \frac{4}{5}i = a+bi$

$\therefore -\frac{1}{5} + \frac{4}{5}i = a+bi \quad \therefore a = -\frac{1}{5}, b = \frac{4}{5}$

$\therefore 9a^2 + b^2 = 9\left(-\frac{1}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = 1$

Answers of exercise 2

First Multiple choice questions

- | | | | | |
|--------|--------|--------|--------|--------|
| (1) a | (2) c | (3) a | (4) c | (5) c |
| (6) c | (7) b | (8) b | (9) d | (10) b |
| (11) c | (12) d | (13) c | (14) d | (15) d |
| (16) d | (17) b | (18) d | (19) d | (20) d |

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Second

Essay questions

1

$$(1) \text{ Discriminant} = (-2)^2 - 4 \times 1 \times 5 = -16 < 0$$

$\therefore$ The two roots are complex and not real number.

$$(2) \text{ Discriminant} = (-10)^2 - 4 \times 1 \times 25 = 0$$

$\therefore$ The two roots are real and equal.

$$(3) \text{ Discriminant} = (5)^2 - 4(-1)(-30) = -95 < 0$$

$\therefore$ The two roots are complex and not real numbers.

$$(4) \because x - 11 - x^2 + 6x = 0$$

$$\therefore x^2 - 7x + 11 = 0$$

$$\therefore \text{The discriminant} = (-7)^2 - 4 \times 1 \times 11 = 5 > 0$$

$\therefore$ The two roots are real and different.

$$(5) \because x - \frac{2}{x-1} = 4 \quad \text{multiplying by } (x-1)$$

$$\therefore x^2 - x - 2 = 4x - 4 \quad \therefore x^2 - 5x + 2 = 0$$

$$\therefore \text{The discriminant} = (-5)^2 - 4 \times 1 \times 2 = 17 > 0$$

$\therefore$ The two roots are real and different.

$$(6) \because (x-1)(x-7) = 2(x-3)(x-4)$$

$$\therefore x^2 - 8x + 7 = 2x^2 - 14x + 24$$

$$\therefore x^2 - 6x + 17 = 0$$

$$\therefore \text{The discriminant} = (-6)^2 - 4 \times 1 \times 17 = -32 < 0$$

$\therefore$ The two roots are complex and not real.

2

$$\because \text{The discriminant} = (-3)^2 - 4 \times 2 \times 2 = -7 < 0$$

$\therefore$ The two roots are complex and not real.

$$\therefore x = \frac{3 \pm \sqrt{-7}}{4} = \frac{3 \pm \sqrt{7}i}{4}$$

$$\therefore \text{The two roots are : } \frac{3 + \sqrt{7}i}{4}, \frac{3 - \sqrt{7}i}{4}$$

3

$$(1) \because \text{The two roots are equal}$$

$$\therefore \text{The discriminant} = 0$$

$$\therefore (-3)^2 - 4 \times 1 \times \left(2 + \frac{1}{k}\right) = 0$$

$$\therefore 8 + \frac{4}{k} = 9 \quad \therefore \frac{4}{k} = 1 \quad \therefore k = 4$$

$$(2) \because \text{The two roots are equal}$$

$$\therefore \text{Discriminant} = \text{zero}$$

$$\therefore (2k+3)^2 - 4 \times 1 \times k^2 = 0$$

$$\therefore 4k^2 + 12k + 9 - 4k^2 = 0$$

$$\therefore k = -\frac{3}{4}$$

$$(3) \because \text{The two roots are equal}$$

$$\therefore \text{The discriminant} = 0$$

$$\therefore [2(k-1)]^2 - 4 \times 1 \times (2k+1) = 0$$

$$\therefore 4k^2 - 8k + 4 - 8k - 4 = 0$$

$$\therefore 4k^2 - 16k = 0$$

$$\therefore 4k(k-4) = 0 \quad \therefore k = 0 \text{ or } k = 4$$

$$\because \text{The two roots are equal and each of them}$$

$$= \frac{-2k+2 \pm \sqrt{0}}{2 \times 1} = \frac{-2k+2}{2} = -k+1$$

$$\therefore \text{At } k = 0, \text{ then the two roots are equal and each of them} = 1$$

$$\text{At } k = 4, \text{ then the two roots are equal and each of them} = -3$$

$$(4) \because \text{The equation is : } x^2 - (2k+6)x + (7k+9) = 0$$

$$\because \text{The two roots are equal}$$

$$\therefore \text{The discriminant} = 0$$

$$\therefore (2k+6)^2 - 4 \times 1 \times (7k+9) = 0$$

$$\therefore 4k^2 + 24k + 36 - 28k - 36 = 0$$

$$\therefore 4k^2 - 4k = 0 \quad \therefore 4k(k-1) = 0$$

$$\therefore k = 0 \text{ or } k = 1$$

$$\because \text{The two roots are equal and each of them}$$

$$= \frac{(2k+6) \pm \sqrt{0}}{2 \times 1} = k+3$$

$$\therefore \text{At } k = 0$$

$$\therefore \text{then the two roots are equal and each of them} = 3$$

$$\therefore \text{at } k = 1$$

$$\therefore \text{then the two roots are equal and each of them} = 4$$

4

$$\text{The discriminant} = (-2m)^2 - 4 \times (m-1) \times m$$

$$= 4m^2 - 4m^2 + 4m = 4m$$

$$\therefore \text{the equation has no real roots}$$

$$\therefore 4m < 0 \quad \therefore m < 0 \quad \therefore m \in]-\infty, 0[$$

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5

∴ The coefficients are rational numbers

$$\begin{aligned}\therefore \text{The discriminant} &= (L - M)^2 - 4 \times L \times -M \\ &= L^2 - 2LM + M^2 + 4LM \\ &= L^2 + 2LM + M^2 = (L + M)^2 \text{ (a perfect square)}\end{aligned}$$

∴ The two roots are rational.

6

$$\therefore (X - a)(X - b) = 5$$

$$\therefore X^2 - (a + b)X + ab - 5 = 0$$

$$\begin{aligned}\therefore \text{The discriminant} &= (a + b)^2 - 4 \times 1 \times (ab - 5) \\ &= a^2 + 2ab + b^2 - 4ab + 20 \\ &= a^2 - 2ab + b^2 + 20 \\ &= (a - b)^2 + 20\end{aligned}$$

is a positive quantity for all the real values of a, b

∴ The two roots are real and different.

Higher skills

1 (1) d (2) c

Instructions to solve 1:

$$(1) \therefore \text{The discriminant} = (-2\sqrt{5})^2 - 4(1)(1)$$

∴ The roots are real

∴ the coefficient of "X" is irrational number.

∴ The roots are real but irrational

$$(2) \therefore (b^2 - 4ac) \text{ is not positive}$$

∴ Either $(b^2 - 4ac)$ is negative and so the roots of the equation are complex and conjugate or $(b^2 - 4ac) = \text{zero}$

∴ The roots are real and equal

∴ $a, b, c \in \mathbb{R}$

∴ The roots are complex and conjugate.

2

$$\begin{aligned}\text{The discriminant} &= (2a)^2 - 4 \times 1 \times (a^2 - b^2 - c^2) \\ &= 4a^2 - 4a^2 + 4b^2 + 4c^2 \\ &= 4(b^2 + c^2) \geq 0\end{aligned}$$

(for every real value of b, c)

∴ The two roots are real.

Answers of Exercise 3

First

Multiple choice questions

- | | | | | |
|--------|--------|--------|--------|--------|
| (1) d | (2) c | (3) d | (4) c | (5) c |
| (6) b | (7) c | (8) b | (9) d | (10) d |
| (11) d | (12) a | (13) c | (14) a | (15) a |
| (16) b | (17) d | (18) c | (19) b | (20) c |

Second

Essay questions

1

$$(1) \therefore 3X^2 - 23X + 30 = 0$$

∴ The sum of the two roots = $\frac{23}{3}$
∴ their product = 10

$$(2) \therefore 4X^2 + 25X + 6 = 3X^2 - 10X + 8$$

$$\therefore X^2 + 35X - 2 = 0$$

∴ The sum of the two roots = -35
∴ their product = -2

(3) Multiplying both sides by L.C.M. of denominators which is $2X$

$$\therefore X^2 + 2 = 3X \quad \therefore X^2 - 3X + 2 = 0$$

∴ The sum of the two roots = 3

∴ their product = 2

$$(4) \therefore (a - 1)X^2 + (1 - a^2)X + a - 1 = 0$$

∴ The sum of the two roots

$$= \frac{-(1 - a^2)}{a - 1} = \frac{a^2 - 1}{a - 1} = \frac{(a - 1)(a + 1)}{a - 1} = a + 1$$

∴ their product = $\frac{a - 1}{a - 1} = 1$

2

∴ The product of the two roots = $\frac{c}{a}$

$$\therefore \frac{-c}{3} = \frac{-8}{3} \quad \therefore c = 8$$

$$\therefore 3X^2 + 10X - 8 = 0 \quad \therefore (3X - 2)(X + 4) = 0$$

$$\therefore X = \frac{2}{3} \text{ or } X = -4$$

3

∴ The sum of the two roots = $\frac{-b}{a}$

$$\therefore \frac{-b}{2} = \frac{-3}{2} \quad \therefore b = 3$$

$$\therefore 2X^2 + 3X - 5 = 0$$

$$\therefore (2X + 5)(X - 1) = 0 \quad \therefore X = \frac{-5}{2} \text{ or } X = 1$$

4

(1) ∴ The sum of the two roots = $\frac{-\text{coefficient of } X}{\text{coefficient of } X^2} = 2$

$$\therefore -1 + \text{the other root} = 2$$

$$\therefore \text{The other root} = 3$$

∴ The product of the two roots

$$= \frac{\text{the absolute term}}{\text{coefficient of } X^2} = a$$

$$\therefore -1 \times 3 = a \quad \therefore a = -3$$

(2) ∴ The sum of the two roots = $\frac{-\text{coefficient of } X}{\text{coefficient of } X^2} = 2$

$$\therefore (1 + i) + \text{the other root} = 2$$

$$\therefore \text{The other root} = 1 - i$$

∴ The product of the two roots

$$= \frac{\text{the absolute term}}{\text{coefficient of } X^2} = a$$

$$\therefore (1 + i)(1 - i) = a$$

$$\therefore 1 - i^2 = a \quad \therefore a = 2$$

5

(1) ∴ The sum of the two roots = $\frac{-\text{coefficient of } X}{\text{coefficient of } X^2} = -a$

$$\therefore 2 + 5 = -a$$

$$\therefore a = -7$$

∴ The product of the two roots

$$= \frac{\text{the absolute term}}{\text{coefficient of } X^2} = b$$

$$\therefore 2 \times 5 = b$$

$$\therefore b = 10$$

(2) ∴ The product of the two roots = $\frac{\text{the absolute term}}{\text{coefficient of } X^2}$

$$= \frac{-21}{a}$$

$$\therefore -3 \times 7 = \frac{-21}{a}$$

$$\therefore a = 1$$

SIMPLEST MATHS BOOKLETS

$\therefore$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = b$

$$\therefore -3 + 7 = b \quad \therefore b = 4$$

(3) $\therefore$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = \frac{1}{a}$

$$\therefore -1 + \frac{3}{2} = \frac{1}{a} \quad \therefore a = 2$$

$\therefore$ The product of the two roots

$$= \frac{\text{the absolute term}}{\text{coefficient of } x^2} = \frac{b}{2}$$

$$\therefore -1 \times \frac{3}{2} = \frac{b}{2} \quad \therefore b = -3$$

(4) $\therefore$ The sum of the two roots $= \frac{-\text{coefficient of } x}{\text{coefficient of } x^2} = -a$

$$\therefore \sqrt{3}i + (-\sqrt{3}i) = -a \quad \therefore a = 0$$

$\therefore$ The product of the two roots

$$= \frac{\text{the absolute term}}{\text{coefficient of } x^2} = b$$

$$\therefore \sqrt{3}i \times (-\sqrt{3}i) = b \quad \therefore b = 3$$

6

(1) $\therefore$ One of the two roots is the additive inverse of the other.

$$\therefore k - 1 = 0 \quad \therefore k = 1$$

(2) $\therefore$ One of the two roots is the multiplicative inverse of the other.

$$\therefore 4k = k^2 + 4 \quad \therefore k^2 - 4k + 4 = 0$$

$$\therefore (k - 2)^2 = 0 \quad \therefore k = 2$$

(3) $\therefore 2x^2 - 5x + k^2 - 2 = 0$

One of the two roots is the multiplicative inverse of the other.

$$\therefore k^2 - 2 = 2 \quad \therefore k = \pm 2$$

Third Higher skills

1 (1) c (2) b (3) a (4) c

Instructions to solve **1** :

(1) $\therefore$ The coefficients are real numbers and one of the two roots is $2i$, then the other root is $-2i$

$\therefore$ sum of the two roots $= 2i - 2i = \text{zero}$

$\therefore$ product of the two roots $= 2i \times (-2i)$

$$= -4 \times -1 = 4$$

and the discriminant < 0

(2) $\therefore b, c$ are real numbers.

$\therefore$ If one of the roots is $(3 + i)$, then the other root $(3 - i)$ and that is sufficient to find b and c

2

$\therefore$ Discriminant $= (2a - 1)^2 - 12(a - 4)$

$$= 4a^2 - 4a + 1 - 12a + 48$$

$$= 4a^2 - 16a + 49$$

$$= 4(a^2 - 4a + 4) + 33$$

$$= 4(a - 2)^2 + 33 > 0$$

whatever the value of a

$\therefore$ This equation has two different roots and these roots have different signs if the product of the roots < 0

$$\therefore \frac{a-4}{3} < 0$$

$$\therefore a - 4 < 0$$

$$\therefore a < 4$$

$$\therefore a \in]-\infty, 4[$$

Answers of Exercise 4

First Multiple choice questions

(1) d (2) c (3) b (4) b (5) a

(6) a (7) d (8) d (9) b (10) c

(11) c (12) c (13) b (14) a (15) b

(16) d (17) b

SIMPLEST MATHS BOOKLETS

Second Essay questions

1

- (1) $\therefore$ The sum of the two roots = 2
 $\therefore$ their product = -8
 $\therefore$ The equation is : $x^2 - 2x - 8 = 0$
- (2) $\therefore$ The sum of the two roots = 14
 $\therefore$ their product = 49
 $\therefore$ The equation is : $x^2 - 14x + 49 = 0$
- (3) $\therefore$ The sum of the two roots = -7
 $\therefore$ their product = 0
 $\therefore$ The equation is : $x^2 + 7x = 0$
- (4) $\therefore$ The sum of the two roots = $\frac{13}{6}$
 $\therefore$ their product = 1
 $\therefore$ The equation is : $x^2 - \frac{13}{6}x + 1 = 0$
i.e. $6x^2 - 13x + 6 = 0$
- (5) $\therefore$ The sum of the two roots = $-\frac{8}{5}$
 $\therefore$ their product = $-\frac{33}{25}$
 $\therefore$ The equation is : $x^2 - \frac{8}{5}x + \frac{33}{25} = 0$
i.e. $25x^2 + 40x - 33 = 0$
- (6) $\therefore$ The sum of the two roots = $3\sqrt{3}$
 $\therefore$ their product = -30
 $\therefore$ The equation is : $x^2 - 3\sqrt{3}x - 30 = 0$
- (7) $\therefore$ The sum of the two roots = 14
 $\therefore$ their product = 29
 $\therefore$ The equation is : $x^2 - 14x + 29 = 0$
- (8) $\therefore$ The sum of the two roots = 0
 $\therefore$ their product = 25
 $\therefore$ The equation is : $x^2 + 25 = 0$
- (9) $\therefore$ The sum of the two roots = 2
 $\therefore$ their product = 10
 $\therefore$ The equation is : $x^2 - 2x + 10 = 0$
- (10) $\therefore$ The sum of the two roots = 6
 $\therefore$ their product = 17
 $\therefore$ The equation is : $x^2 - 6x + 17 = 0$
- (11) $\therefore$ The sum of the two roots = $\frac{3}{1} + \frac{3+3i}{1-i}$
 $= \frac{3-3i+3i-3}{1+i} = 0$
 $\therefore$ their product = $\frac{3}{1} \times \frac{3+3i}{1-i} = \frac{9+9i}{1+i} = 9$
 $\therefore$ The equation is : $x^2 + 9 = 0$
- (12) $\therefore$ The sum of the two roots
 $= \frac{-2+2i}{1+i} + \frac{-2-4i}{2-i} = \frac{-2+6i-6i+2}{3+i} = 0$
 $\therefore$ their product = $\frac{-2+2i}{1+i} \times \frac{-2-4i}{2-i} = \frac{12+4i}{3+i} = 4$
 $\therefore$ The equation is : $x^2 + 4 = 0$

2

$$\begin{aligned} & \boxed{L+M=7}, \boxed{LM=5} \\ (1) & L^2M + M^2L = LM(L+M) \\ & = 5 \times 7 = 35 \\ (2) & \frac{1}{M} + \frac{1}{L} = \frac{L+M}{LM} = \frac{7}{5} \\ (3) & (L-2)(M-2) = LM - 2(L+M) + 4 \\ & = 5 - 14 + 4 = -5 \\ (4) & \left(L + \frac{1}{M}\right) \left(M + \frac{1}{L}\right) = LM + 2 + \frac{1}{LM} \\ & = 5 + 2 + \frac{1}{5} = 7\frac{1}{5} \end{aligned}$$

3

$$\begin{aligned} & \therefore L+M=3, LM=-5 \\ & \therefore \text{let } D, E \text{ be the two roots of the required equation} \\ & \therefore D=L-4, E=M-4 \\ & \therefore D+E=L-4+M-4=(L+M)-8 \\ & = 3-8=-5 \\ & \therefore DE=(L-4)(M-4)=LM-4(M+L)+16 \\ & = -5-4(3)+16=-1 \\ & \therefore \text{The required equation is : } x^2+5x-1=0 \end{aligned}$$

4

$$\begin{aligned} & \text{let the two roots of the given equation be : } L, M \\ & \text{and the two roots of the required equation be : } D, E \\ & \therefore D=L+1, E=M+1 \\ & \therefore L=D-1 \quad (1) \\ & \therefore L \text{ is one of the roots of the equation : } \\ & x^2-7x-9=0 \quad \therefore L^2-7L-9=0 \\ & \therefore \text{from (1) : } \therefore (D-1)^2-7(D-1)-9=0 \\ & \therefore D^2-2D+1-7D+7-9=0 \\ & \therefore D^2-9D-1=0 \\ & \therefore D \text{ is a root of the equation : } \\ & x^2-9x-1=0 \text{ which is the required equation.} \end{aligned}$$

5

$$\begin{aligned} & \text{let the two roots of the given equation be : } L, M \\ & \therefore L+M=-3, LM=-5 \\ & \therefore \text{let the two roots of the required equation be : } D, E \\ & \therefore D=L^2, E=M^2 \\ & \therefore D+E=L^2+M^2=(L+M)^2-2LM \\ & = 9+10=19 \\ & \therefore DE=(L^2M^2)=(LM)^2=(-5)^2=25 \\ & \therefore \text{The required equation is : } x^2-19x+25=0 \end{aligned}$$

11

$$\begin{aligned} & \therefore L+M=\frac{3}{2}, LM=-\frac{1}{2} \\ & \therefore \text{let the two roots of the required equation be : } D, E \\ & \therefore D=\frac{L}{M}, E=\frac{M}{L} \\ & \therefore D+E=\frac{L}{M}+\frac{M}{L}=\frac{L^2+M^2}{LM}=\frac{(L+M)^2-2LM}{LM} \\ & = \frac{\left(\frac{3}{2}\right)^2-2\times-\frac{1}{2}}{-\frac{1}{2}} = \frac{-13}{-\frac{1}{2}} = \frac{26}{1} \\ & \therefore DE=\frac{L}{M} \times \frac{M}{L} = 1 \\ & \therefore \text{The required equation is : } x^2 + \frac{13}{2}x + 1 = 0 \\ & \text{i.e. } 2x^2 + 13x + 2 = 0 \end{aligned}$$

6

$$\begin{aligned} & \text{let the two roots of the first equation be : } L, M \\ & \therefore L-M=\pm\sqrt{k^2-8k} \\ & \therefore \text{let the two roots of the second equation be : } D, E \\ & \therefore DE=k \\ & \therefore L-M=2DE \quad \therefore \pm\sqrt{k^2-8k}=2k \\ & \therefore \text{by squaring both sides :} \\ & \therefore k^2-8k=4k^2 \quad \therefore 3k^2+8k=0 \\ & \therefore k(3k+8)=0 \quad \therefore k=0 \text{ or } k=-\frac{8}{3} \end{aligned}$$

SIMPLEST MATHS BOOKLETS

Third Higher skills

(1) d (2) b (3) b

Instructions to solve :

(1) Let the roots of the equation (the rectangle dimensions) be L and M

$$\therefore LM = 15 \quad , \quad 2(L + M) = 26$$

$$\therefore L + M = 13$$

$$\therefore \text{The quadratic equation is : } x^2 - 13x + 15 = 0$$

(2) $\therefore a^2 + 3a + 1 = 0 \quad , \quad b^2 + 3b + 1 = 0$

$\therefore a$ and b are the roots of the equation :

$$x^2 + 3x + 1 = 0$$

$$\therefore a + b = -3 \quad , \quad ab = 1$$

$$\therefore \frac{a}{b} + \frac{b}{a} = \frac{a^2 + b^2}{ab} = \frac{(a+b)^2 - 2ab}{ab} = \frac{(-3)^2 - 2(1)}{(1)} = 7$$

(3) $\therefore (x-a)(x-b) = k$

$$\therefore x^2 - (a+b)x + ab - k = 0$$

$$\therefore L + M = a + b \quad , \quad LM = ab - k$$

and so $ab = LM + k$

$\therefore$ The quadratic equation whose roots are a, b is $x^2 - (L + M)x + LM + k = 0$

$$\therefore (x-L)(x-M) + k = 0$$

Answers of Exercise 5

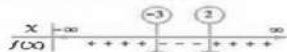
First Multiple choice questions

- (1) c (2) a (3) b (4) d (5) a
 (6) d (7) a (8) d (9) b (10) b
 (11) c (12) b (13) d (14) c (15) c
 (16) d (17) b (18) a

Second Essay questions

(1) $\therefore f(x) = (x-2)(x+3)$

$\therefore$ The roots of the equation :
 $f(x) = 0$ are $x = 2, x = -3$



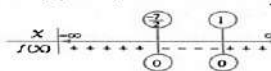
- f is positive at $x \in \mathbb{R} - [-3, 2]$
- $f(x) = 0$ at $x \in \{-3, 2\}$
- f is negative at $x \in]-3, 2[$

(2) $\therefore f(x) = (2x-3)^2 \quad \therefore$ when $f(x) = 0$
 $\therefore (2x-3)^2 = 0 \quad \therefore x = \frac{3}{2}$
 $\therefore a = 4 > 0$



$\therefore f$ is positive $\forall x \in \mathbb{R} - \{\frac{3}{2}\}$

(3) $\therefore f(x) = 2x^2 + 5x - 7$
 when $f(x) = 0 \quad \therefore 2x^2 + 5x - 7 = 0$
 $\therefore (2x+7)(x-1) = 0 \quad \therefore x = -\frac{7}{2}$ or $x = 1$

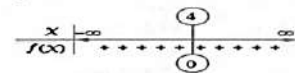


- The sign of f is the same as a (where $a = 2 > 0$)
 thus f is positive at $x \in \mathbb{R} - [-\frac{7}{2}, 1]$
- $f(x) = 0$ at $x \in \{-\frac{7}{2}, 1\}$
- The sign of f is negative at $x \in]-\frac{7}{2}, 1[$

(4) $\therefore f(x) = x^2 - 8x + 16$

$$\text{when } f(x) = 0 \quad \therefore x^2 - 8x + 16 = 0$$

$$\therefore (x-4)^2 = 0 \quad \therefore x = 4$$



$\therefore$ The sign of f is the same as a (where $a = 1 > 0$)

$\therefore f$ is positive at $x \in \mathbb{R} - \{4\}$

$\therefore f(x) = 0$ at $x = 4$

(5) $\therefore f(x) = 2x^2 - 3x + 5$

$$\therefore \text{The discriminant} = b^2 - 4ac = (-3)^2 - 4 \times 2 \times 5 = 9 - 40 = -31 < 0$$

$\therefore$ There's no real zeroes to the function.

$\therefore$ The equation has no real roots.

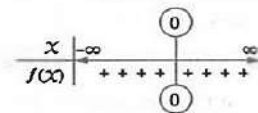
$\therefore a$ (coefficient of x^2) $= 2 > 0$



$\therefore f$ is positive $\forall x \in \mathbb{R}$

(6) $\therefore f(x) = 2x^2 \quad \therefore f(x) = 0$ at $x = 0$

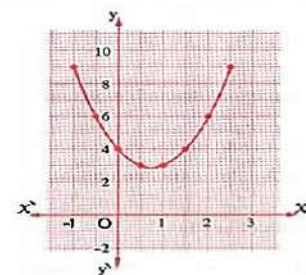
$\therefore a$ (coefficient of x^2) $= 2 > 0$



$\therefore f$ is positive $\forall x \in \mathbb{R} - \{0\}$

2 $f(x) = 2x^2 - 3x + 4$

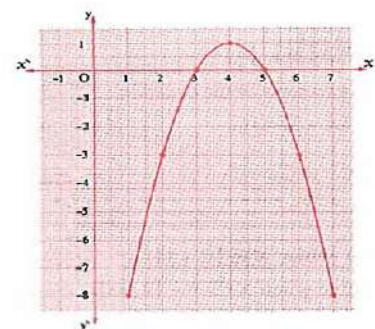
x	-1	$-\frac{1}{2}$	0	$\frac{1}{2}$	1	$1\frac{1}{2}$	2	$2\frac{1}{2}$
$f(x)$	9	6	4	3	3	4	6	9



From the graph : f is positive $\forall x \in \mathbb{R}$

3 $f(x) = -x^2 + 8x - 15$

x	1	2	3	4	5	6	7
$f(x)$	-8	-3	0	1	0	-3	-8



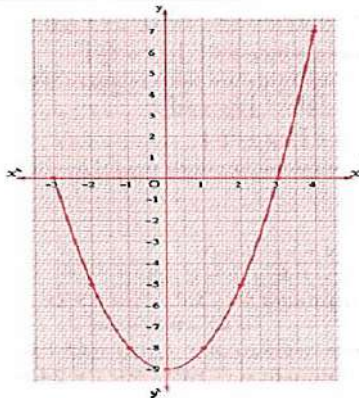
From the graph , we get :

- $f(x) = 0$ at $x \in \{3, 5\}$
- f is negative at $x \in \mathbb{R} - [3, 5]$
- f is positive at $x \in]3, 5[$
- $\therefore$ The S.S. of the equation : $f(x) = 0$ is $\{3, 5\}$

SIMPLEST MATHS BOOKLETS

4 $f(x) = x^2 - 9$

x	-3	-2	-1	0	1	2	3	4
f(x)	0	-5	-8	-9	-8	-5	0	7



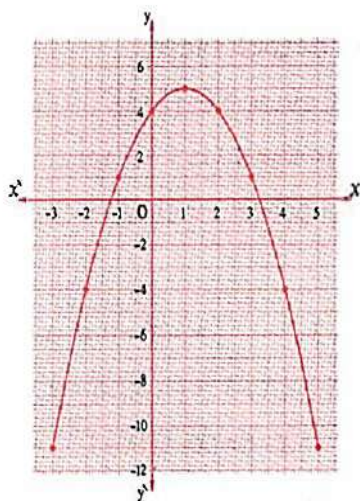
From the graph, we get :

- f is negative at $x \in]-3, 3[$
- $f(x) = 0$ at $x \in \{-3, 3\}$
- f is positive at $x \in]3, 4[$

5

$f(x) = -x^2 + 2x + 4$

x	-3	-2	-1	0	1	2	3	4	5
f(x)	-11	-4	1	4	5	4	1	-4	-11



From the graph, we get :

- $f(x) = 0$ at $x \in \{-1.2, 3.2\}$
- f is positive at $x \in [-1.2, 3.2]$
- f is negative at $x \in [-3, -1.2[\cup]3.2, 5]$

Notice that : 3.2, -1.2 are approximated values for the roots of the equation related to the function.

6

$\therefore 2x^2 - kx + k - 3 = 0$

$\therefore a = 2, b = -k, c = k - 3$

$\therefore$ The discriminant $= (-k)^2 - 4 \times 2 \times (k - 3)$
 $= k^2 - 8k + 24$

$\therefore$ Investigate the sign of $f : f(k) = k^2 - 8k + 24$

$\therefore$ The discriminant $= (-8)^2 - 4 \times 1 \times 24 = -32 < 0$

$\therefore$ The equation : $k^2 - 8k + 24 = 0$

its two roots are not real numbers

$\therefore$ the coefficient of $k^2 > 0$

$\therefore$ The sign of f is positive for all values of $k \in \mathbb{R}$

$\therefore$ The discriminant of the equation :

$2x^2 - kx + k - 3 = 0$ (positive for all values $x \in \mathbb{R}$)

$\therefore$ The two roots are real and different for all $x \in \mathbb{R}$

7

(1) From the graph, we get :

- $f(x) = 0$ at $x \in \{-1, 5\}$
- f is negative at $x \in \mathbb{R} - [-1, 5]$
- f is positive at $x \in]-1, 5[$

(2) From the graph, we get :

- $f(x) = 0$ at $x \in \{1, 3\}$
- f is positive at $x \in \mathbb{R} - [1, 3]$
- f is negative at $x \in]1, 3[$

Third Higher skills

(1) From the graph :

- f is positive when $x \in \mathbb{R} - [-3, 2]$
- $f(x) = 0$ when $x \in \{-3, 2\}$
- f is negative when $x \in]-3, 2[$

To find the rule of the function :

$\therefore f(x) = a(x - 2)(x + 3)$

$\therefore$ The curve passes through the point $(0, -6)$

$\therefore -6 = a \times -2 \times 3 \quad \therefore a = 1$

$\therefore f(x) = (x - 2)(x + 3) = x^2 + x - 6$

(2) From the graph :

- f is negative when $x \in \mathbb{R} - [-3, 0]$
- $f(x) = 0$ when $x \in \{-3, 0\}$
- f is positive when $x \in]-3, 0[$

To find the rule of the function :

$\therefore f(x) = a \times (x + 3)$

$\therefore$ The curve passes through the point $(-1, 2)$

$\therefore 2 = -a(-1 + 3) \quad \therefore a = -1$

$\therefore f(x) = -x(x + 3) = -x^2 - 3x$

(3) From the graph :

- f is positive when $x \in \mathbb{R} - [1, 5]$
- $f(x) = 0$ when $x \in \{1, 5\}$
- f is negative when $x \in]1, 5[$

$\therefore f(x) = a(x - 1)(x - 5)$

$\therefore$ The curve passes through the point $(3, -4)$

$\therefore -4 = a(3 - 1)(3 - 5) \quad \therefore -4 = a \times 2 \times -2$

$\therefore a = 1$

$\therefore f(x) = (x - 1)(x - 5) = x^2 - 6x + 5$

SIMPLEST MATHS BOOKLETS

Answers of Exercise 6

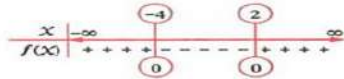
First Multiple choice questions

- (1) b (2) c (3) d (4) c (5) d
 (6) d (7) c (8) b (9) d (10) c
 (11) c (12) a (13) b (14) b (15) c
 (16) c (17) a (18) d

Second Essay questions

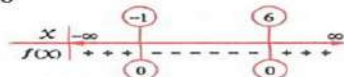
1

(1) $f(x) = x^2 + 2x - 8$ let $x^2 + 2x - 8 = 0$
 $\therefore (x+4)(x-2) = 0 \therefore x = -4$ or $x = 2$
 $\therefore a > 0$



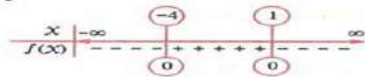
$\therefore f$ is positive when $x \in \mathbb{R} - [-4, 2]$
 $\therefore \text{S.S.} = \mathbb{R} - [-4, 2]$

(2) $f(x) = x^2 - 5x - 6$ let $x^2 - 5x - 6 = 0$
 $\therefore (x+1)(x-6) = 0 \therefore x = -1$ or $x = 6$
 $\therefore a > 0$



$\therefore f$ is negative when $x \in]-1, 6[$
 $\therefore \text{S.S.} =]-1, 6[$

(3) $f(x) = 4 - 3x - x^2$ let $4 - 3x - x^2 = 0$
 $\therefore x^2 + 3x - 4 = 0 \therefore (x+4)(x-1) = 0$
 $\therefore x = -4$ or $x = 1$
 $\therefore a < 0$



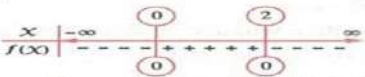
$\therefore f$ is positive when $x \in]-4, 1[$
 $\therefore f(x) = 0$ when $x \in \{-4, 1\}$
 $\therefore \text{S.S.} = [-4, 1]$

(4) $f(x) = x^2 - 1$ let $x^2 - 1 = 0$
 $\therefore (x+1)(x-1) = 0 \therefore x = -1$ or $x = 1$
 $\therefore a > 0$



$\therefore f$ is negative when $x \in]-1, 1[$
 $\therefore f(x) = 0$ when $x \in \{-1, 1\}$
 $\therefore \text{S.S.} = [-1, 1]$

(5) $f(x) = 2x - x^2$ let $2x - x^2 = 0$
 $\therefore x(2-x) = 0 \therefore x = 0$ or $x = 2$
 $\therefore a < 0$



$\therefore f$ is negative when $x \in \mathbb{R} - [0, 2]$
 $\therefore \text{S.S.} = \mathbb{R} - [0, 2]$

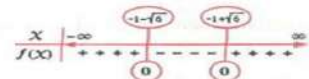
(6) $f(x) = x^2 - 4x + 4$ let $x^2 - 4x + 4 = 0$
 $\therefore (x-2)^2 = 0 \therefore x = 2$
 $\therefore a > 0$



$\therefore f$ is positive when $x \in \mathbb{R} - \{2\}$
 $\therefore f(x) = 0$ when $x = 2 \therefore \text{S.S.} = \mathbb{R}$

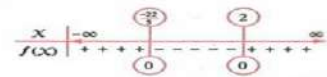
2

(1) $\therefore 5 - 2x \leq x^2 \therefore x^2 + 2x - 5 \geq 0$
 $\therefore f(x) = x^2 + 2x - 5$ let $x^2 + 2x - 5 = 0$
 $\therefore x = \frac{-2 \pm \sqrt{(2)^2 - 4 \times 1 \times (-5)}}{2 \times 1} = \frac{-2 \pm \sqrt{24}}{2} = -1 \pm \sqrt{6}$
 $\therefore x = -1 + \sqrt{6}$ or $x = -1 - \sqrt{6}$
 $\therefore a > 0$



$\therefore f$ is positive when $x \in \mathbb{R} - [-1 - \sqrt{6}, -1 + \sqrt{6}]$
 $\therefore f(x) = 0$ at $x \in \{-1 - \sqrt{6}, -1 + \sqrt{6}\}$
 $\therefore \text{S.S.} = \mathbb{R} - [-1 - \sqrt{6}, -1 + \sqrt{6}]$

(2) $\therefore 5x^2 + 12x - 44 \geq 0$
 $\therefore 5x^2 + 12x - 44 \geq 0$
 $\therefore f(x) = 5x^2 + 12x - 44$
 $\therefore 5x^2 + 12x - 44 = 0$
 $\therefore (5x+22)(x-2) = 0 \therefore x = -\frac{22}{5}$ or $x = 2$
 $\therefore a > 0$



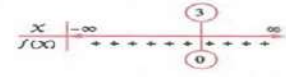
$\therefore f$ is positive when $x \in \mathbb{R} - [-\frac{22}{5}, 2]$
 $\therefore f(x) = 0$ when $x \in \{-\frac{22}{5}, 2\}$
 $\therefore \text{S.S.} = \mathbb{R} - [-\frac{22}{5}, 2]$

(3) $\therefore 3x^2 \leq 11x + 4 \therefore 3x^2 - 11x - 4 \leq 0$
 $\therefore f(x) = 3x^2 - 11x - 4$
 $\therefore 3x^2 - 11x - 4 = 0$
 $\therefore (3x+1)(x-4) = 0 \therefore x = -\frac{1}{3}$ or $x = 4$
 $\therefore a > 0$



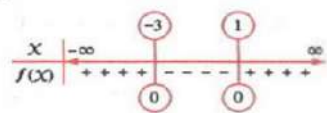
$\therefore f$ is negative when $x \in]-\frac{1}{3}, 4[$
 $\therefore f(x) = 0$ when $x \in \{-\frac{1}{3}, 4\}$
 $\therefore \text{S.S.} = [-\frac{1}{3}, 4]$

(4) $\therefore x^2 \geq 6x - 9 \therefore x^2 - 6x + 9 \geq 0$
 $\therefore f(x) = x^2 - 6x + 9$ let $x^2 - 6x + 9 = 0$
 $\therefore (x-3)^2 = 0 \therefore x = 3$
 $\therefore a > 0$



$\therefore f$ is positive when $x \in \mathbb{R} - \{3\} \therefore \text{S.S.} = \mathbb{R}$

(5) $\therefore x(x+2) - 3 \leq 0 \therefore x^2 + 2x - 3 \leq 0$
 $\therefore f(x) = x^2 + 2x - 3$ let $x^2 + 2x - 3 = 0$
 $\therefore (x+3)(x-1) = 0 \therefore x = -3$ or $x = 1$
 $\therefore a > 0$



$\therefore f$ is negative at $x \in]-3, 1[$
 $\therefore f(x) = 0$ when $x \in \{-3, 1\}$
 $\therefore \text{S.S.} = [-3, 1]$

(6) $\therefore x^2 + 5 \leq 1 \therefore x^2 + 4 \leq 0$
 $\therefore f(x) = x^2 + 4$ let $x^2 + 4 = 0$
 $\therefore \text{The discriminant} = b^2 - 4ac$
 $= 0 - 4 \times 1 \times 4 = -16 < 0$
 $\therefore \text{The equation has no real roots.}$
 $\therefore a > 0 \therefore f$ is positive $\forall x \in \mathbb{R}$
 $\therefore \text{S.S.} = \emptyset$

SIMPLEST MATHS BOOKLETS

(7) $\because (x-2)^2 \leq -5 \quad \therefore x^2 - 4x + 4 \leq -5$

$\therefore x^2 - 4x + 9 \leq 0$

$\therefore f(x) = x^2 - 4x + 9 \quad \text{let } x^2 - 4x + 9 = 0$

$\therefore \text{The discriminant} = b^2 - 4ac$

$\therefore \text{The equation has no real roots.}$

$\therefore a > 0$

$\therefore f \text{ is positive } \forall x \in \mathbb{R}$

$\therefore \text{S.S.} = \emptyset$

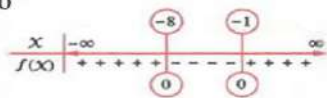
(8) $\because (x+3)^2 < 10 - 3(x+3)$

$\therefore x^2 + 6x + 9 < 1 - 3x \quad \therefore x^2 + 9x + 8 < 0$

$\therefore f(x) = x^2 + 9x + 8 \quad \text{Let } x^2 + 9x + 8 = 0$

$\therefore (x+8)(x+1) = 0 \quad \therefore x = -8 \text{ or } x = -1$

$\therefore a > 0$



$\therefore f \text{ is negative at } x \in]-8, -1[$

$\therefore \text{S.S.} =]-8, -1[$

Answers of Exercise 7

First Multiple choice questions

- | | | | |
|--------|--------|--------|--------|
| (1) b | (2) d | (3) c | (4) c |
| (5) c | (6) d | (7) b | (8) b |
| (9) a | (10) b | (11) c | (12) d |
| (13) c | (14) b | (15) c | (16) c |
| (17) c | (18) c | | |

Second Essay questions

1

- (1) The directed angle isn't in standard position, because the vertex angle isn't the origin point.
- (2) The directed angle isn't in standard position, because its initial side doesn't lie on $\overrightarrow{OX}$
- (3) The directed angle is in standard position.
- (4) The directed angle is in standard position.
- (5) The directed angle isn't in standard position, because the vertex angle isn't the origin point.
- (6) The directed angle isn't in standard position, because its initial side doesn't lie on $\overrightarrow{OX}$

2

(1) -306°

(2) 270°

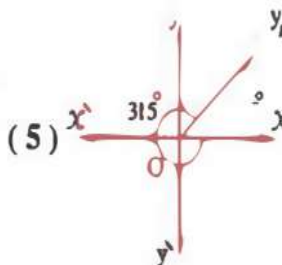
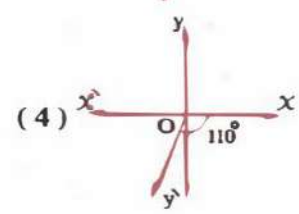
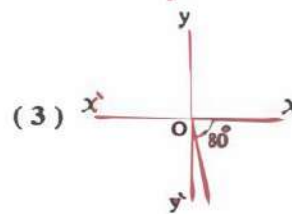
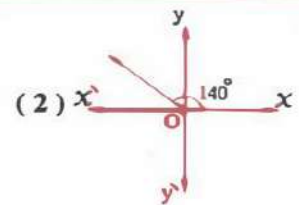
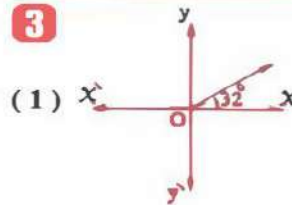
(3) 225°

(4) 300° 28

(5) 245°

(6) -69° 20

3



4

- (1) first (2) third (3) fourth
(4) second

5

- (1) 304° , fourth (2) 240° , third
(3) 145° , second (4) 220° , third

6

- (1) 400° , -320° (2) 510° , -210°
(3) 235° , -485° (4) 120° , -600°
(5) 180° , -540°

SIMPLEST MATHS BOOKLETS

Answers of Exercise 8

First Multiple choice questions

- (1) b (2) c (3) d (4) b
 (5) d (6) a (7) c (8) d
 (9) b (10) b (11) b (12) b
 (13) b (14) c (15) b (16) c
 (17) c

Second Essay questions

1

$$\theta^{\text{rad}} = x^{\circ} \times \frac{\pi}{180^{\circ}}$$

(1) $\theta^{\text{rad}} = \frac{135^{\circ}}{180^{\circ}} \pi = \frac{3}{4} \pi$
 (2) $\theta^{\text{rad}} = \frac{90^{\circ}}{180^{\circ}} \pi = \frac{1}{2} \pi$
 (3) $\theta^{\text{rad}} = \frac{300^{\circ}}{180^{\circ}} \pi = \frac{5}{3} \pi$
 (4) $\theta^{\text{rad}} = \frac{-235^{\circ}}{180^{\circ}} \pi = -\frac{47}{36} \pi$
 (5) $\theta^{\text{rad}} = \frac{-210^{\circ}}{180^{\circ}} \pi = -\frac{7}{6} \pi$
 (6) $\theta^{\text{rad}} = \frac{112.5^{\circ}}{180^{\circ}} \pi = \frac{5}{8} \pi$
 (7) $\theta^{\text{rad}} = \frac{390^{\circ}}{180^{\circ}} \pi = \frac{13}{6} \pi$
 (8) $\theta^{\text{rad}} = \frac{780^{\circ}}{180^{\circ}} \pi = \frac{13}{3} \pi$

2

$$\theta^{\text{rad}} = x^{\circ} \times \frac{\pi}{180^{\circ}}$$

(1) $\theta^{\text{rad}} = 58^{\circ} \times \frac{\pi}{180^{\circ}} \approx 1.012^{\text{rad}}$
 (2) $\theta^{\text{rad}} = 56.6^{\circ} \times \frac{\pi}{180^{\circ}} \approx 0.988^{\text{rad}}$
 (3) $\theta^{\text{rad}} = 37^{\circ} 15' \times \frac{\pi}{180^{\circ}} \approx 0.650^{\text{rad}}$
 (4) $\theta^{\text{rad}} = 115^{\circ} 38' 6'' \times \frac{\pi}{180^{\circ}} \approx 2.018^{\text{rad}}$
 (5) $\theta^{\text{rad}} = 257^{\circ} 54' \times \frac{\pi}{180^{\circ}} \approx 4.486^{\text{rad}}$
 (6) $\theta^{\text{rad}} = 160^{\circ} 50' 48'' \times \frac{\pi}{180^{\circ}} \approx 2.807^{\text{rad}}$

3

$$x^{\circ} = \theta^{\text{rad}} \times \frac{180^{\circ}}{\pi}$$

(1) $x^{\circ} = \frac{11}{15} \times 180^{\circ} = 132^{\circ}$
 (2) $x^{\circ} = 0.72 \times 180^{\circ} = 129^{\circ} 36'$
 (3) $x^{\circ} = 0.49 \times \frac{180^{\circ}}{\pi} = 28^{\circ} 4' 30''$
 (4) $x^{\circ} = -1.67 \times \frac{180^{\circ}}{\pi} = -95^{\circ} 41' 2''$
 (5) $x^{\circ} = 2.27 \times \frac{180^{\circ}}{\pi} = 130^{\circ} 3' 41''$
 (6) $x^{\circ} = -3 \frac{1}{2} \times \frac{180^{\circ}}{\pi} = -200^{\circ} 32' 7''$

4

$\therefore$ The measure of the inscribed angle $= 45^{\circ}$
 $\therefore$ The measure of the central angle subtended by the same arc $= 2 \times 45^{\circ} = 90^{\circ}$
 $\therefore \theta^{\text{rad}} = 90^{\circ} \times \frac{\pi}{180^{\circ}} = \frac{\pi}{2}$
 $\therefore r = \frac{L}{\theta^{\text{rad}}} = 12 \div \frac{\pi}{2} = \frac{24}{\pi} \text{ cm.}$
 $\therefore$ The circumference $= 2 \pi r = 2 \pi \times \frac{24}{\pi} = 48 \text{ cm.}$

5

The area of $\triangle AMB = \frac{1}{2} \times AM \times BM$
 $\therefore AM = BM = r \quad \therefore \frac{1}{2} r^2 = 32$
 $\therefore r^2 = 64 \quad \therefore r = 8 \text{ cm.}$
 $\therefore$ Length of $\widehat{AB} = 90^{\circ} \times \frac{\pi}{180^{\circ}} \times 8 = 12.57 \text{ cm.}$
 $\therefore$ The perimeter of the shaded part
 $= 8 + 8 + 12.57 = 28.57 \text{ cm.}$

6

Const. : Draw $\overline{AM}$

Proof : $\therefore \overline{AB}, \overline{AC}$

are two tangents

to the circle M.

$\therefore \overline{MB} \perp \overline{AB}, \overline{MC} \perp \overline{AC}$

$\therefore m(\angle M) = 360^{\circ} - (90^{\circ} + 90^{\circ} + 60^{\circ}) = 120^{\circ}$

$\therefore m(\text{reflex } M) = 360^{\circ} - 120^{\circ} = 240^{\circ}$

$\therefore \overline{AM}$ bisects $\angle A \quad \therefore m(\angle BAM) = 30^{\circ}$

$\therefore MB = \frac{1}{2} AM$

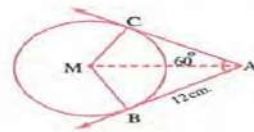
$\therefore MB = r \quad \therefore AM = 2r$

In $\triangle ABM$ is right-angled at B

$\therefore (2r)^2 = r^2 + (12)^2 \quad \therefore 3r^2 = 144$

$\therefore r^2 = 48 \quad \therefore r = 4\sqrt{3} \text{ cm.}$

$\therefore$ Length of greater $\widehat{BC} = 240^{\circ} \times \frac{\pi}{180^{\circ}} \times 4\sqrt{3} = 29 \text{ cm.}$



Third Higher skills

1

(1) b (2) d (3) b

Instructions to solve 1 :

(1) The length of the arc $= \theta^{\text{rad}} r = \frac{72^{\circ}}{180^{\circ}} \times \pi \times 14$
 $= \frac{28}{5} \pi \text{ cm.}$

$\therefore$ The circumference of the circle $= \frac{28}{5} \pi$
 $\therefore 2 \pi r = \frac{28}{5} \pi$

$\therefore r = \frac{14}{5} = 2.8 \text{ cm.}$

(2) $\therefore 5 < \text{the length of arc } \widehat{AB} < 6$

$\therefore 5 < \frac{x}{180^{\circ}} \times \pi \times 10 < 6$

$\therefore 5 < \frac{\pi x}{180^{\circ}} < 6 \quad \therefore 28.6^{\circ} < x < 34.4^{\circ}$

(3) $\therefore$ The ratio between measures of angles of the quadrilateral $= 5 : 4 : 9 : 6$

$\therefore 5x + 4x + 9x + 6x = 360^{\circ}$

$\therefore 24x = 360^{\circ} \quad \therefore x = 15^{\circ}$

$\therefore$ Measure of the smallest angle in the quadrilateral $= 4 \times 15^{\circ} = 60^{\circ}$

$\therefore$ in radian measure $= 60^{\circ} \times \frac{\pi}{180^{\circ}} = \frac{\pi}{3}$

2

The degree measure of the angle which the straight line makes with the X-axis $= \frac{180^{\circ}}{3} = 60^{\circ}$

$\therefore$ The slope of the straight line $= \tan 60^{\circ} = \sqrt{3}$

$\therefore$ The equation of the straight line $: y = \sqrt{3}x + c$

$\therefore$ The angle in the standard position.

$\therefore c = 0 \quad \therefore y = \sqrt{3}x$

SIMPLEST MATHS BOOKLETS

Answers of Exercise 9

First Multiple choice questions

- (1) a (2) d (3) d (4) b (5) d
 (6) c (7) b (8) a (9) d (10) c
 (11) a (12) d (13) c (14) d (15) c
 (16) a (17) c (18) d (19) c (20) d

Second Essay questions

1

- (1) $\therefore 270^\circ < 350^\circ < 360^\circ$
 $\therefore 350^\circ$ lies in the fourth quad.
 $\therefore \cos 350^\circ$ is positive.
 (2) $\therefore 180^\circ < 265^\circ < 270^\circ \therefore 265^\circ$ lies in 3rd quad.
 $\therefore \sec 265^\circ$ is negative.
 (3) $\therefore \frac{5\pi}{4} = \frac{5 \times 180^\circ}{4} = 225^\circ$
 and it lies in 3rd quad. $\therefore \sin \frac{5\pi}{4}$ is negative.
 (4) $\therefore \frac{3\pi}{7} = \frac{3 \times 180^\circ}{7} = 77^\circ \frac{1}{7}$
 and it lies in 1st quad.
 $\therefore \csc \frac{3\pi}{7}$ is positive.
 (5) $\therefore \tan 410^\circ = \tan (50^\circ + 360^\circ) = \tan 50^\circ$
 $\therefore 50^\circ$ lies in 1st quad.
 $\therefore \tan 410^\circ$ is positive.
 (6) $\therefore \cos (-165^\circ) = \cos (-165^\circ + 360^\circ)$
 $= \cos 195^\circ$
 $\therefore 195^\circ$ lies in 3rd quad.
 $\therefore \cos (-165^\circ)$ is negative.

2

(1) $\therefore x = \frac{2}{3}, y = \frac{\sqrt{5}}{3}$
 $\therefore \sin \theta = \frac{\sqrt{5}}{3}, \cos \theta = \frac{2}{3}$
 $\therefore \tan \theta = \frac{\sqrt{5}}{2}, \cot \theta = \frac{2}{\sqrt{5}}$
 $\therefore \csc \theta = \frac{3}{\sqrt{5}}, \sec \theta = \frac{3}{2}$

(2) $\therefore x = \frac{-3}{5}, y = \frac{-4}{5}$
 $\therefore \sin \theta = \frac{-4}{5}, \cos \theta = \frac{-3}{5}$
 $\therefore \tan \theta = \frac{4}{3}, \cot \theta = \frac{3}{4}$
 $\therefore \csc \theta = \frac{-5}{4}, \sec \theta = \frac{-5}{3}$

(3) $\therefore x = 0, y = -1$
 $\therefore \sin \theta = -1, \cos \theta = 0$
 $\therefore \tan \theta$ is undefined, $\cot \theta = 0$
 $\therefore \csc \theta = -1, \sec \theta$ is undefined.

3

(1) $\therefore x^2 + y^2 = 1 \therefore (0.6)^2 + y^2 = 1$
 $\therefore y^2 = 0.64 \therefore y = 0.8$ such that $y > 0$
 $\therefore B(0.6, 0.8)$
 $\therefore \cos \theta = 0.6, \sin \theta = 0.8,$
 $\tan \theta = \frac{4}{3}, \sec \theta = \frac{5}{3}, \csc \theta = \frac{5}{4}, \cot \theta = \frac{3}{4}$

(2) $\therefore x^2 + y^2 = 1$

$\therefore x^2 + (-0.6)^2 = 1 \therefore x^2 = 0.64$
 $\therefore x = 0.8$ such that $x > 0 \therefore B(0.8, -0.6)$
 $\therefore \cos \theta = 0.8, \sin \theta = -0.6$
 $\therefore \tan \theta = \frac{-3}{4}, \sec \theta = \frac{5}{4}$
 $\therefore \csc \theta = \frac{-5}{3}, \cot \theta = \frac{-4}{3}$

(3) $\therefore x^2 + y^2 = 1 \therefore \frac{3}{4} + y^2 = 1$

$\therefore y^2 = \frac{1}{4}$
 $\therefore y = \frac{1}{2}$ such that $90^\circ < \theta < 180^\circ$
 $\therefore B\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$
 $\therefore \cos \theta = -\frac{\sqrt{3}}{2}, \sin \theta = \frac{1}{2}, \tan \theta = \frac{-1}{\sqrt{3}}$
 $\therefore \sec \theta = \frac{-2}{\sqrt{3}}, \csc \theta = 2, \cot \theta = -\sqrt{3}$

(4) $\therefore x^2 + y^2 = 1 \therefore x^2 + \frac{5}{9} = 1$

$\therefore x^2 = \frac{4}{9} \therefore x = \frac{-2}{3} \therefore x < 0$
 $\therefore B\left(-\frac{2}{3}, \frac{\sqrt{5}}{3}\right)$
 $\therefore \cos \theta = \frac{-2}{3}, \sin \theta = \frac{\sqrt{5}}{3}, \tan \theta = \frac{-\sqrt{5}}{2}$
 $\therefore \sec \theta = \frac{-3}{2}, \csc \theta = \frac{3}{\sqrt{5}}, \cot \theta = \frac{-2}{\sqrt{5}}$

(5) $\therefore x^2 + y^2 = 1 \therefore 1 + y^2 = 1$

$\therefore y^2 = 0 \therefore y = 0$
 $\therefore B(-1, 0)$
 $\therefore \cos \theta = -1, \sin \theta = 0, \tan \theta = 0$
 $\therefore \sec \theta = -1, \csc \theta$ is undefined, $\cot \theta$ is undefined.

(6) $\therefore x^2 + y^2 = 1 \therefore (-x^2) + x^2 = 1$

$\therefore 2x^2 = 1 \therefore x = \frac{1}{\sqrt{2}} \therefore x > 0$
 $\therefore B\left(\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$
 $\therefore \cos \theta = \frac{1}{\sqrt{2}}, \sin \theta = \frac{1}{\sqrt{2}}, \tan \theta = 1$
 $\therefore \sec \theta = \sqrt{2}, \csc \theta = \sqrt{2}, \cot \theta = 1$

4

(1) $\tan 0^\circ + \tan 45^\circ + \tan 180^\circ = 0 + 1 + 0 = 1$

(2) $\sin 180^\circ \cos 45^\circ - \cos 180^\circ \sin 45^\circ$
 $= 0 \times \frac{1}{\sqrt{2}} - (-1) \times \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{2}}$

(3) $\sec \frac{\pi}{6} \tan \frac{\pi}{3} - \cot \frac{\pi}{3} \cos \frac{\pi}{6}$
 $= \sec 30^\circ \tan 60^\circ - \cot 60^\circ \cos 30^\circ$
 $= \frac{2}{\sqrt{3}} \times \sqrt{3} - \frac{1}{\sqrt{3}} \times \frac{\sqrt{3}}{2} = 2 - \frac{1}{2} = \frac{3}{2}$

(4) $\frac{4 \sin^2 30^\circ - 3 \tan 45^\circ \cos 0^\circ}{2 \cos 60^\circ + 2 \sin 45^\circ \cos 45^\circ}$
 $= \frac{4 \times \left(\frac{1}{2}\right)^2 - 3 \times 1 \times 1}{2 \times \frac{1}{2} + 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}} = \frac{1-3}{1+1} = -1$

(5) $3 \sin 30^\circ \sin^2 60^\circ - \cos 0^\circ \sec 60^\circ$
 $+ \sin 270^\circ \cos^2 45^\circ$
 $= 3 \times \frac{1}{2} \times \left(\frac{\sqrt{3}}{2}\right)^2 - 1 \times 2 + (-1) \times \left(\frac{1}{\sqrt{2}}\right)^2$
 $= \frac{-11}{8}$

SIMPLEST MATHS BOOKLETS

5

$$(1) \text{ L.H.S} = 2 \times (1)^2 = 2,$$

$$\text{R.H.S} = -2 \times (-1) = 2 \quad \therefore \text{L.H.S} = \text{R.H.S}$$

$$(2) \text{ L.H.S} = 3 \times \frac{\sqrt{3}}{2} \times \sqrt{3} - 2 \times \sqrt{2} \times \sqrt{2}$$

$$= \frac{9}{2} - 4 = \frac{1}{2} = \text{R.H.S}$$

$$(3) \text{ L.H.S} = 3(1)^2 - 2 \times \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2}$$

$$= 3 - \frac{3}{2} = \frac{3}{2}$$

$$\text{R.H.S} = \frac{3}{2} \times 1 = \frac{3}{2} \quad \therefore \text{L.H.S} = \text{R.H.S.}$$

$$(4) \text{ L.H.S} = \frac{2}{\sqrt{3}} \times \sqrt{3} + \left(\frac{2}{\sqrt{3}}\right)^2 - (1)^2$$

$$= 2 + \frac{4}{3} - 1 = \frac{7}{3} = \text{R.H.S}$$

$$(5) \text{ L.H.S} = \frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2} \times \frac{1}{2} = \frac{1}{2},$$

$$\text{R.H.S} = \sin^2 45^\circ = \left(\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}$$

$$\therefore \text{L.H.S} = \text{R.H.S}$$

6

$$(1) \because X \sin^2 45^\circ \cos 180^\circ = \tan^2 60^\circ \sin 270^\circ$$

$$\therefore X \left(\frac{1}{\sqrt{2}}\right)^2 \times (-1) = (\sqrt{3})^2 \times (-1)$$

$$\therefore -\frac{1}{2} X = -3 \quad \therefore X = 6$$

$$(2) \because X \sin 45^\circ \cos 45^\circ \cot 30^\circ = \tan^2 45^\circ - \cos^2 60^\circ$$

$$\therefore X \left(\frac{1}{\sqrt{2}}\right) \times \left(\frac{1}{\sqrt{2}}\right) \times (\sqrt{3}) = (1)^2 - \left(\frac{1}{2}\right)^2$$

$$\therefore \frac{\sqrt{3}}{2} X = \frac{3}{4} \quad \therefore X = \frac{\sqrt{3}}{2}$$

Answers of Exercise 10

First Multiple choice questions

- (1) c (2) b (3) b (4) b (5) b
 (6) b (7) b (8) c (9) b (10) d
 (11) d (12) a (13) a (14) c (15) a
 (16) c (17) d (18) b (19) b (20) c

Second Essay questions

1

$$(1) \sin 150^\circ = \sin (180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2}$$

$$(2) \sec 210^\circ = \sec (180^\circ + 30^\circ) = -\sec 30^\circ = \frac{-2}{\sqrt{3}}$$

$$(3) \tan 240^\circ = \tan (180^\circ + 60^\circ) = \tan 60^\circ = \sqrt{3}$$

$$(4) \cos (-150^\circ) = \cos 150^\circ$$

$$= \cos (180^\circ - 30^\circ) = -\cos 30^\circ = -\frac{\sqrt{3}}{2}$$

$$(5) \tan 225^\circ = \tan (180^\circ + 45^\circ) = \tan 45^\circ = 1$$

$$(6) \csc \frac{11\pi}{6} = \csc \left(\frac{12\pi}{6} - \frac{\pi}{6}\right) = -\csc \frac{\pi}{6} = -2$$

$$(7) \cot 780^\circ = \cot (720^\circ + 60^\circ) = \cot 60^\circ = \frac{1}{\sqrt{3}}$$

$$(8) \cos (-900^\circ) = \cos (-900^\circ + 3 \times 360^\circ)$$

$$= \cos 180^\circ = -1$$

2

$$(1) \cos 120^\circ + \tan 225^\circ + \csc 330^\circ + \cos 420^\circ$$

$$= \cos (180^\circ - 60^\circ) + \tan (180^\circ + 45^\circ)$$

$$+ \csc (360^\circ - 30^\circ) + \cos (360^\circ + 60^\circ)$$

$$= -\cos 60^\circ + \tan 45^\circ - \csc 30^\circ + \cos 60^\circ$$

$$= -\frac{1}{2} + 1 - 2 + \frac{1}{2} = -1$$

$$(2) \because \cos 930^\circ = \cos (2 \times 360^\circ + 210^\circ)$$

$$= \cos 210^\circ = \cos (180^\circ + 30^\circ) = -\cos 30^\circ$$

$$\therefore \sin 150^\circ \cos (-300^\circ) - \cos 30^\circ \cot 240^\circ$$

$$= \sin (180^\circ - 30^\circ) \cos (360^\circ - 60^\circ)$$

$$- \cos 30^\circ \cot (180^\circ + 60^\circ)$$

$$= \sin 30^\circ \cos 60^\circ - \cos 30^\circ \cot 60^\circ$$

$$= \frac{1}{2} \times \frac{1}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{\sqrt{3}} = -\frac{1}{4}$$

$$(3) \tan \frac{2\pi}{3} \sec \frac{11\pi}{3} + \cot \frac{11\pi}{6} \csc \frac{19\pi}{6}$$

$$+ \tan \frac{25\pi}{6} \csc \left(\frac{-19\pi}{3}\right)$$

$$= \tan \left(\pi - \frac{\pi}{3}\right) \sec \left(\frac{12\pi}{3} - \frac{\pi}{3}\right)$$

$$+ \cot \left(\frac{12\pi}{6} - \frac{\pi}{6}\right) \csc \left(\frac{12\pi}{6} + \frac{7\pi}{6}\right)$$

$$+ \tan \left(\frac{24\pi}{6} + \frac{\pi}{6}\right) \csc \left(\frac{-18\pi}{3} - \frac{\pi}{3}\right)$$

$$= -\tan \frac{\pi}{3} \sec \frac{\pi}{3} - \cot \left(\frac{\pi}{6}\right) \csc \left(\pi + \frac{1}{6}\pi\right)$$

$$- \tan \frac{\pi}{6} \csc \left(\frac{\pi}{3}\right)$$

$$= -\tan \frac{\pi}{3} \sec \frac{\pi}{3} + \cot \frac{\pi}{6} \csc \frac{\pi}{6} - \tan \frac{\pi}{6} \csc \frac{\pi}{3}$$

$$= -\sqrt{3} \times 2 + \sqrt{3} \times 2 - \frac{1}{\sqrt{3}} \times \frac{2}{\sqrt{3}} = -\frac{2}{3}$$

3

$$(1) \cos (-300^\circ) = \cos 300^\circ = \cos (360^\circ - 60^\circ)$$

$$= \cos 60^\circ = \frac{1}{2}$$

$$\therefore \sin 420^\circ = \sin (60^\circ + 360^\circ) = \sin 60^\circ = \frac{\sqrt{3}}{2}$$

$$\therefore \cos 750^\circ = \cos (30^\circ + 360^\circ \times 2)$$

$$= \cos 30^\circ = \frac{\sqrt{3}}{2},$$

$$\cos 660^\circ = \cos (300^\circ + 360^\circ) = \cos 300^\circ$$

$$= \cos (360^\circ - 60^\circ) = \cos 60^\circ = \frac{1}{2}$$

$$\therefore \text{L.H.S} = \frac{1}{2} \times \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2} \times \frac{1}{2} = 0 = \text{R.H.S}$$

SIMPLEST MATHS BOOKLETS

$$\begin{aligned}
 (2) \sin 600^\circ &= \sin (360^\circ + 240^\circ) = \sin 240^\circ \\
 &= \sin (180^\circ + 60^\circ) = -\sin 60^\circ = -\frac{\sqrt{3}}{2} \\
 \therefore \cos (-30^\circ) &= \cos 30^\circ = \frac{\sqrt{3}}{2} \\
 \therefore \sin 150^\circ &= \sin (180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2} \\
 \therefore \cos (-240^\circ) &= \cos 240^\circ = \cos (180^\circ + 60^\circ) \\
 &= -\cos 60^\circ = -\frac{1}{2} \\
 \therefore \text{L.H.S} &= -\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} + \frac{1}{2} \times \left(-\frac{1}{2}\right) \\
 &= -1 = \text{R.H.S}
 \end{aligned}$$

$$\begin{aligned}
 (3) \sin 150^\circ &= \sin (180^\circ - 30^\circ) = \sin 30^\circ = \frac{1}{2} \\
 \therefore \tan 225^\circ &= \tan (180^\circ + 45^\circ) = \tan 45^\circ = 1 \\
 \therefore \cos 315^\circ &= \cos (360^\circ - 45^\circ) = \cos 45^\circ = \frac{1}{\sqrt{2}} \\
 \therefore \sec (-120^\circ) &= \sec 120^\circ = \sec (180^\circ - 60^\circ) \\
 &= -\sec 60^\circ = -2 \\
 \therefore \sin (-135^\circ) &= -\sin 135^\circ = -\sin (180^\circ - 45^\circ) \\
 &= -\sin 45^\circ = -\frac{1}{\sqrt{2}} \\
 \therefore \csc 210^\circ &= \csc (180^\circ + 30^\circ) \\
 &= -\csc 30^\circ = -2 \\
 \therefore \text{L.H.S} &= \frac{1}{2} \times 1 + \frac{1}{\sqrt{2}} \times (-2) + \left(-\frac{1}{\sqrt{2}}\right) \times (-2) \\
 &= \frac{1}{2} - \frac{2}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{1}{2} = \text{R.H.S}
 \end{aligned}$$

$$\begin{aligned}
 (4) \sin \theta &= \frac{4}{5}, \cos \theta = -\frac{3}{5} \\
 (1) \sin (180^\circ + \theta) &= -\sin \theta = -\frac{4}{5} \\
 (2) \cos \left(\frac{\pi}{2} - \theta\right) &= \cos (90^\circ - \theta) = \sin \theta = \frac{4}{5} \\
 (3) \tan (360^\circ - \theta) &= -\tan \theta = -\frac{\frac{4}{5}}{\frac{-3}{5}} = \frac{4}{3} \\
 (4) \csc \left(\frac{3\pi}{2} - \theta\right) &= \csc (270^\circ - \theta) = -\sec \theta = \frac{5}{3} \\
 (5) \sec (\theta + \pi) &= \sec (\theta + 180^\circ) = -\sec \theta = \frac{5}{3} \\
 (6) \sin (\theta - \pi) &= \sin (\theta - 180^\circ) = \sin (180^\circ + \theta) \\
 &= -\sin \theta = -\frac{4}{5}
 \end{aligned}$$

$$\begin{aligned}
 (5) \sin \theta &= \frac{2}{3}, \cos \theta = \frac{\sqrt{5}}{3} \\
 (1) \sin (270^\circ + \theta) &= -\cos \theta = -\frac{\sqrt{5}}{3} \\
 (2) \sec (270^\circ + \theta) &= \csc \theta = \frac{3}{2} \\
 (3) \csc \left(\theta + \frac{\pi}{2}\right) &= \csc (\theta + 90^\circ) = \sec \theta = \frac{3}{\sqrt{5}}
 \end{aligned}$$

$$\begin{aligned}
 (6) (1) \therefore \sin (3\theta + 15^\circ) &= \cos (2\theta - 5^\circ) \\
 \therefore 3\theta + 15^\circ + 2\theta - 5^\circ &= 90^\circ \\
 \therefore 5\theta + 10^\circ &= 90^\circ \\
 \therefore 5\theta &= 80^\circ \quad \therefore \theta = 16^\circ \\
 (2) \therefore \sec (\theta + 25^\circ) &= \csc (\theta + 15^\circ) \\
 \therefore \theta + 25^\circ + \theta + 15^\circ &= 90^\circ \\
 \therefore 2\theta + 40^\circ &= 90^\circ \quad \therefore 2\theta = 50^\circ \quad \therefore \theta = 25^\circ \\
 (3) \therefore \tan (\theta + 20^\circ) &= \cot (3\theta + 30^\circ) \\
 \therefore \theta + 20^\circ + 3\theta + 30^\circ &= 90^\circ \\
 \therefore 4\theta + 50^\circ &= 90^\circ \quad \therefore 4\theta = 40^\circ \quad \therefore \theta = 10^\circ \\
 (4) \therefore \cos \left(\frac{\theta + 20^\circ}{2}\right) &= \sin \left(\frac{\theta + 40^\circ}{2}\right) \\
 \therefore \frac{\theta + 20^\circ}{2} + \frac{\theta + 40^\circ}{2} &= 90^\circ \\
 \therefore \theta + 20^\circ + \theta + 40^\circ &= 180^\circ \\
 \therefore 2\theta + 60^\circ &= 180^\circ \\
 \therefore 2\theta &= 120^\circ \quad \therefore \theta = 60^\circ
 \end{aligned}$$

$$\begin{aligned}
 (7) \therefore \sin 2\theta &= \cos \theta \\
 \therefore 2\theta \pm \theta &= \frac{\pi}{2} + 2\pi n \text{ where } n \in \mathbb{Z} \\
 \text{either } 2\theta + \theta &= \frac{\pi}{2} + 2\pi n \\
 \therefore 3\theta &= \frac{\pi}{2} + 2\pi n \quad \therefore \theta = \frac{\pi}{6} + \frac{2\pi}{3}n \\
 \text{or } 2\theta - \theta &= \frac{\pi}{2} + 2\pi n \\
 \therefore \theta &= \frac{\pi}{2} + 2\pi n \\
 \therefore \text{The solution is : } &\frac{\pi}{6} + \frac{2\pi}{3}n \text{ or } \frac{\pi}{2} + 2\pi n \\
 (2) \therefore \cos 5\theta &= \sin \theta \quad \therefore \sin \theta = \cos 5\theta \\
 \therefore \theta \pm 5\theta &= \frac{\pi}{2} + 2\pi n \\
 \text{either } \theta + 5\theta &= \frac{\pi}{2} + 2\pi n \\
 \therefore 6\theta &= \frac{\pi}{2} + 2\pi n \quad \therefore \theta = \frac{\pi}{12} + \frac{\pi}{3}n \\
 \text{or } \theta - 5\theta &= \frac{\pi}{2} + 2\pi n \\
 \therefore -4\theta &= \frac{\pi}{2} + 2\pi n \quad \therefore \theta = -\frac{\pi}{8} - \frac{\pi}{2}n \\
 \therefore \text{The solution is : } &\frac{\pi}{12} + \frac{\pi}{3}n \text{ or } -\frac{\pi}{8} - \frac{\pi}{2}n
 \end{aligned}$$

$$\begin{aligned}
 (8) (1) \therefore \csc (\theta + 15^\circ) &= \sec 42^\circ \\
 \therefore (\theta + 15^\circ) \pm (42^\circ) &= 90^\circ + 360^\circ n \\
 \therefore \theta + 15^\circ + 42^\circ &= 90^\circ \quad \therefore \theta = 33^\circ \\
 (2) \therefore \sin (\theta + 30^\circ) &= \cos \theta \\
 \therefore (\theta + 30^\circ) \pm \theta &= 90^\circ + 360^\circ n \\
 \therefore \theta + 30^\circ + \theta &= 90^\circ \\
 \therefore 2\theta &= 60^\circ \quad \therefore \theta = 30^\circ
 \end{aligned}$$

$$\begin{aligned}
 (3) \therefore \sin \theta &= \cos \theta \\
 \therefore \theta \pm \theta &= 90^\circ + 360^\circ n \\
 \therefore 2\theta &= 90^\circ \quad \therefore \theta = 45^\circ \\
 (4) \therefore \csc \left(\theta - \frac{\pi}{6}\right) &= \sec \theta \\
 \therefore \left(\theta - \frac{\pi}{6}\right) \pm \theta &= 90^\circ + 360^\circ n \\
 \therefore 2\theta - 30^\circ &= 90^\circ \\
 \therefore 2\theta &= 120^\circ \quad \therefore \theta = 60^\circ
 \end{aligned}$$

$$\begin{aligned}
 (9) (1) \therefore \tan \theta - 1 &= 0 \quad \therefore \tan \theta = 1 \\
 \therefore \tan \text{ is positive in the first and third quad.} \\
 \therefore \theta &= 45^\circ \text{ or } \theta = 180^\circ + 45^\circ = 225^\circ \\
 \therefore \theta &\in]0, \frac{\pi}{2}[\quad \therefore \theta = 45^\circ \\
 (2) \therefore 2 \cos \theta - 1 &= 0 \quad \therefore \cos \theta = \frac{1}{2} \\
 \therefore \cos \text{ is positive in the first and fourth quad.} \\
 \therefore \theta &= 60^\circ \text{ or } \theta = 360^\circ - 60^\circ = 300^\circ \\
 \therefore \theta &\in]0, \frac{\pi}{2}[\quad \therefore \theta = 60^\circ \\
 (3) \therefore 2 \cos \left(\frac{\pi}{2} - \theta\right) &= 1 \quad \therefore \cos \left(\frac{\pi}{2} - \theta\right) = \frac{1}{2} \\
 \therefore \sin \theta &= \frac{1}{2} \\
 \therefore \sin \text{ is positive in the first and the second quad.} \\
 \therefore \theta &= 30^\circ \text{ or } \theta = 180^\circ - 30^\circ = 150^\circ \\
 \therefore \theta &\in]0, \frac{\pi}{2}[\quad \therefore \theta = 30^\circ \\
 (4) \therefore 2 \sin \left(\frac{\pi}{2} - \theta\right) &= \sqrt{3} \\
 \therefore \sin (90^\circ - \theta) &= \frac{\sqrt{3}}{2} \quad \therefore \cos \theta = \frac{\sqrt{3}}{2} \\
 \therefore \cos \text{ is positive in the first and fourth quad.} \\
 \therefore \theta &= 30^\circ \text{ or } 360^\circ - 30^\circ = 330^\circ \\
 \therefore \theta &\in]0, \frac{\pi}{2}[\quad \therefore \theta = 30^\circ
 \end{aligned}$$

$$\begin{aligned}
 (10) \therefore \cos \left(\frac{3\pi}{2} - \theta\right) &= \frac{\sqrt{3}}{2} \quad \therefore \cos (270^\circ - \theta) = \frac{\sqrt{3}}{2} \\
 \therefore -\sin \theta &= \frac{\sqrt{3}}{2} \quad \therefore \sin \theta = -\frac{\sqrt{3}}{2} \\
 \therefore \sin \left(\frac{\pi}{2} + \theta\right) &= \frac{1}{2} \quad \therefore \sin (90^\circ + \theta) = \frac{1}{2} \\
 \therefore \cos \theta &= \frac{1}{2} \\
 \therefore \sin \text{ (negative) and cos (positive)} \\
 \therefore \theta &\text{ lies in 4th quad.} \\
 \therefore \text{The acute angle whose sin} &= \frac{\sqrt{3}}{2} \text{ is } 60^\circ \\
 \therefore \theta &= 360^\circ - 60^\circ = 300^\circ
 \end{aligned}$$

SIMPLEST MATHS BOOKLETS

Answers of Exercise 11

First Multiple choice questions

- (1) b (2) b (3) a (4) b
(5) a (6) d (7) c (8) c

Second Essay questions

1

	max. value	min. value	the range
(1)	$\frac{1}{2}$	$-\frac{1}{2}$	$[-\frac{1}{2}, \frac{1}{2}]$
(2)	$\frac{1}{3}$	$-\frac{1}{3}$	$[-\frac{1}{3}, \frac{1}{3}]$
(3)	2	-2	$[-2, 2]$

2 Form the table and draw by yourself, from the graph we get :

	min. value	max. value	the range
(1)	-4	4	$[-4, 4]$
(2)	-4	4	$[-4, 4]$
(3)	-2	2	$[-2, 2]$
(4)	-3	3	$[-3, 3]$

3

$$(1) \because 0^\circ \leq \theta \leq 120^\circ \quad \therefore 0^\circ \leq 3\theta \leq 360^\circ$$

By giving to 3θ some values to some special angles :

$$0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \frac{4\pi}{6}, \dots, 2\pi$$

$$\therefore \theta = 0, \frac{\pi}{18}, \frac{2\pi}{18}, \frac{3\pi}{18}, \frac{4\pi}{18}, \dots, \frac{12\pi}{18}$$

$$\therefore y = \cos 3\theta$$

form the table, then draw the graph by yourself

from the graph we get the max. value = 1

the min. value = -1 and the range = $[-1, 1]$

$$(2) \because 0^\circ \leq \theta \leq 180^\circ \quad \therefore 0^\circ \leq 2\theta \leq 360^\circ$$

By giving to 2θ some values to some special

$$\text{angles : } 0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}, \dots, 2\pi$$

$$\therefore \theta = 0, \frac{\pi}{12}, \frac{2\pi}{12}, \frac{3\pi}{12}, \dots, \pi$$

$$\therefore y = 5 \sin 2\theta$$

form the table, then draw the graph by yourself

from the graph we get the min. value = -5 the

max. value = 5 and the range = $[-5, 5]$

4

Draw by yourself, from the graph we get the range of $y = 4 \cos \theta$ is $[-4, 4]$

the max. value = 4, the min. value = -4

the range of the function : $y = 3 \sin \theta$ is $[-3, 3]$

the max. value = 3, the min. value = -3

Answers of Exercise 12

First Multiple choice questions

- (1) a (2) c (3) c (4) c (5) b
(6) a (7) a (8) a (9) a (10) c
(11) b (12) d (13) c

Second Essay questions

1

$$(1) 36^\circ 52' 12'' \quad (2) 38^\circ 8' 25''$$

$$(3) 67^\circ 51' 34''$$

$$(4) \because \theta = \tan^{-1}(-0.8227), -0.8227 \text{ negative.}$$

$$\therefore \theta \text{ lies in } 2^{\text{nd}} \text{ or } 4^{\text{th}} \text{ quad.}$$

$$\therefore \theta = 180^\circ - (39^\circ 26' 39'') = 140^\circ 33' 21''$$

$$(5) \because \theta = \sin^{-1}(-0.4652), -0.4652 \text{ (negative)}$$

$$\therefore \theta \text{ lies in } 3^{\text{rd}} \text{ or } 4^{\text{th}} \text{ quad.}$$

$$\therefore \theta = 180^\circ + (27^\circ 43' 23'') = 207^\circ 43' 23''$$

$$(6) \because \theta = \cos^{-1}(-0.5206), -0.5206 \text{ (negative)}$$

$$\therefore \theta \text{ lies in } 2^{\text{nd}} \text{ or } 3^{\text{rd}} \text{ quad.}$$

$$\therefore \theta = 180^\circ - (58^\circ 37' 39'') = 121^\circ 22' 21''$$

$$(7) 15^\circ 26' 7''$$

$$(8) \because \theta = \cot^{-1}(-1.4612), -1.4612 \text{ (negative)}$$

$$\therefore \theta \text{ lies in } 2^{\text{nd}} \text{ or } 4^{\text{th}} \text{ quad.}$$

$$\therefore \theta = 180^\circ - (34^\circ 23' 12'') = 145^\circ 36' 48''$$

$$(9) 17^\circ 22' 23''$$

$$(10) \because \theta = \csc^{-1}(-2.5466), -2.5466 \text{ (negative)}$$

$$\therefore \theta \text{ lies in } 3^{\text{rd}} \text{ or } 4^{\text{th}} \text{ quad.}$$

$$\therefore \theta = 180^\circ + (23^\circ 7' 17'') = 203^\circ 7' 17''$$

$$(11) \because \theta = \sec^{-1}(-3.57), -3.57 \text{ (negative)}$$

$$\therefore \theta \text{ lies in } 2^{\text{nd}} \text{ or } 3^{\text{rd}} \text{ quad.}$$

$$\therefore \theta = 180^\circ - (73^\circ 43' 59'') = 106^\circ 16' 1''$$

$$(12) 19^\circ 35' 59''$$

2

$$(1) \because \theta = \sin^{-1} 0.86603, 0.86603 \text{ (positive)}$$

$$\therefore \theta \text{ lies in } 1^{\text{st}} \text{ or } 2^{\text{nd}} \text{ quad.}$$

$$\therefore \theta = 60^\circ 0' 2'' \text{ or } \theta = 180^\circ - 60^\circ 0' 2'' = 119^\circ 59' 58''$$

$$(2) \because \theta = \cos^{-1}(-0.4752), -0.4752 \text{ (negative)}$$

$$\therefore \theta \text{ lies in } 2^{\text{nd}} \text{ or } 3^{\text{rd}} \text{ quad.}$$

$$\therefore \theta = 180^\circ - (61^\circ 37' 39'') = 118^\circ 22' 21''$$

$$\text{or } \theta = 180^\circ + (61^\circ 37' 39'') = 241^\circ 37' 39''$$

$$(3) \because \theta = \csc^{-1}(-1.2576), -1.2576 \text{ (negative)}$$

$$\therefore \theta \text{ lies in } 3^{\text{rd}} \text{ or } 4^{\text{th}} \text{ quad.}$$

$$\therefore \theta = 180^\circ + (52^\circ 40' 15'') = 232^\circ 40' 15''$$

$$\text{or } \theta = 360^\circ - (52^\circ 40' 15'') = 307^\circ 19' 45''$$

$$(4) \because \theta = \tan^{-1} 1.5417, 1.5417 \text{ (positive)}$$

$$\therefore \theta \text{ lies in } 1^{\text{st}} \text{ or } 3^{\text{rd}} \text{ quad.}$$

$$\therefore \theta = 57^\circ 1' 52''$$

$$\text{or } \theta = 180^\circ + (57^\circ 1' 52'') = 237^\circ 1' 52''$$

SIMPLEST MATHS BOOKLETS

(5) $\therefore \theta = \cos^{-1}(-0.642)$, -0.642 (negative)

$\therefore \theta$ lies in 2nd or 3rd quad.

$\therefore \theta = 180^\circ - (50^\circ 33' 32'') = 129^\circ 56' 28''$

or $\theta = 180^\circ + (50^\circ 33' 32'') = 230^\circ 33' 32''$

(6) $\therefore \theta = \sec^{-1}(2.0515)$, 2.0515 (positive)

$\therefore \theta$ lies in 1st or 4th quad.

$\therefore \theta = 60^\circ 49' 37''$

or $\theta = 360^\circ - (60^\circ 49' 37'') = 299^\circ 10' 23''$

(7) $\therefore \theta = \csc^{-1}(-1.8715)$, -1.8715 (negative)

$\therefore \theta$ lies in 3rd or 4th quad.

$\therefore \theta = 180^\circ + (32^\circ 17' 55'') = 212^\circ 17' 55''$

or $\theta = 360^\circ - (32^\circ 17' 55'') = 327^\circ 42' 5''$

(8) $\therefore \theta = \cot^{-1}(-2.7012)$, -2.7012 (negative)

$\therefore \theta$ lies in 2nd or 4th quad.

$\therefore \theta = 180^\circ - (20^\circ 18' 53'') = 159^\circ 41' 7''$

or $\theta = 360^\circ - (20^\circ 18' 53'') = 339^\circ 41' 7''$

(9) $\therefore \theta = \tan^{-1}(-2.1456)$, -2.1456 (negative)

$\therefore \theta$ lies in 2nd or 4th quad.

$\therefore \theta = 180^\circ - (65^\circ 0' 40'') = 114^\circ 59' 20''$

or $\theta = 360^\circ - (65^\circ 0' 40'') = 294^\circ 59' 20''$

3

(1) $\therefore B\left(\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$ lies in 1st quad.

$\therefore \theta$ lies in the 1st quad.

$\therefore \sin^{-1}\left(\frac{1}{2}\right) = 30^\circ \quad \therefore \theta = 30^\circ$

(2) $\therefore B\left(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ lies in 2nd quad.

$\therefore \theta$ lies in 2nd quad.

$\therefore \sin^{-1}\frac{1}{\sqrt{2}} = 45^\circ \quad \therefore \theta = 180^\circ - 45^\circ = 135^\circ$

(3) $\therefore B\left(\frac{6}{10}, -\frac{8}{10}\right)$ lies in the 4th quad.

$\therefore \theta$ lies in the 4th quad.

$\therefore \cos^{-1}\frac{6}{10} = 53^\circ 7' 48''$

$\therefore \theta = 360^\circ - (53^\circ 7' 48'') = 306^\circ 52' 12''$

4

(1) $\therefore \tan \theta = \frac{8}{5} \quad \therefore \theta = \tan^{-1} \frac{8}{5}$

$\therefore \theta = 57^\circ 59' 41''$

(2) $\therefore \cos \theta = \frac{7}{9} \quad \therefore \theta = \cos^{-1} \frac{7}{9}$

$\therefore \theta = 38^\circ 56' 33''$

(3) $\therefore \sin \theta = \frac{4}{9} \quad \therefore \theta = \sin^{-1} \frac{4}{9}$

$\therefore \theta = 26^\circ 23' 16''$

5

(1) $\therefore 90^\circ \leq \theta \leq 180^\circ$

$\therefore \theta$ lies in 2nd quad.

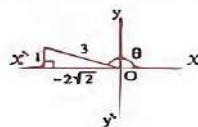
$\therefore \sin \theta = \frac{1}{3}$

$\therefore \theta = \sin^{-1}\left(\frac{1}{3}\right)$

$\therefore \theta = 180^\circ - (19^\circ 28' 16'') = 160^\circ 31' 44''$

(2) $\cos \theta = \frac{-2\sqrt{2}}{3}, \tan \theta = \frac{-1}{2\sqrt{2}}$

$\therefore \sec \theta = \frac{-3}{2\sqrt{2}}$



6

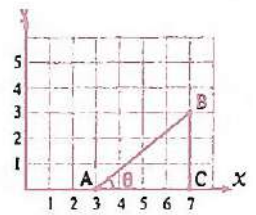
From the graph :

AC = 4 unit length.

BC = 3 unit length.

$\therefore \tan \theta = \frac{3}{4}$

$\therefore \theta = 36^\circ 52' 12''$



Answers to Exercise 1

First Multiple choice questions

- | | | | |
|--------|--------|--------|--------|
| (1) c | (2) b | (3) b | (4) b |
| (5) d | (6) c | (7) c | (8) a |
| (9) c | (10) c | (11) c | (12) d |
| (13) a | | | |

1

$\therefore$ Polygon ABCD $\sim$ polygon EFGH

$\therefore \frac{AB}{EF} = \frac{BC}{FG} = \frac{CD}{GH} = \frac{AD}{EH} = \text{scale factor}$

$\therefore \frac{(y+2)}{6} = \frac{BC}{FG} = \frac{15}{x} = \frac{12}{8}$

$\therefore$ Scale factor = $\frac{12}{8} = \frac{3}{2}$

(First req.)

$\therefore x = 10 \text{ cm}, y + 2 = 9$

$\therefore y = 7 \text{ cm}.$

(Second req.)

2

$\therefore \triangle ADE \sim \triangle ABC$

$\therefore m(\angle ADE) = m(\angle B)$ and they are corresponding angles.

$\therefore \overline{DE} \parallel \overline{BC}$ (First req.)

$\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} \quad \therefore \frac{6}{AB} = \frac{4}{12} = \frac{5}{AC}$

$\therefore AB = 18 \text{ cm}.$

$\therefore BD = 12 \text{ cm}.$

$\therefore AC = 15 \text{ cm}.$

$\therefore CE = 10 \text{ cm}.$ (Second req.)

3

$\therefore \triangle ABC \sim \triangle DEF$

$\therefore \frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{\text{perimeter of } \triangle ABC}{\text{perimeter of } \triangle DEF}$

$\therefore \frac{AB}{8} = \frac{BC}{9} = \frac{AC}{10} = \frac{81}{27}$

$\therefore AB = 24 \text{ cm}, BC = 27 \text{ cm}.$

$\therefore AC = 30 \text{ cm}.$

(The req.)

4

Let the two dimensions of the second rectangle be $x \text{ cm}.$ and $y \text{ cm}.$

$\therefore$ The two rectangles are similar.

$\therefore \frac{8}{x} = \frac{12}{y} = \frac{40}{200}$

$\therefore x = 40 \text{ cm}, y = 60 \text{ cm}.$

$\therefore$ Area of second rectangle = $40 \times 60 = 2400 \text{ cm}^2$

(The req.)

5

$\therefore$ Polygon ABCD $\sim$ polygon XYZL

$\therefore m(\angle A) = m(\angle X) = 115^\circ$

$\therefore m(\angle XLZ) = 360^\circ - (115^\circ + 85^\circ + 70^\circ) = 90^\circ$

$\therefore$ polygon ABCD $\sim$ polygon XYZL

$\therefore \frac{AD}{XL} = \frac{BC}{YZ} = \frac{\text{perimeter of polygon ABCD}}{\text{perimeter of polygon XYZL}}$

$\therefore \frac{AD}{4.8} = \frac{6}{8} = \frac{19.5}{\text{perimeter of polygon XYZL}}$

$\therefore AD = 3.6 \text{ cm}.$

(First req.)

$\therefore$ perimeter of polygon XYZL = $26 \text{ cm}.$ (Second req.)

6

(1) XY

(2) CD

(3) AD

(4) XYZL, ABCD

SIMPLEST MATHS BOOKLETS

7

(1) Notice that the required rectangle is

an enlargement of the given rectangle and let
rectangle $\hat{A}\hat{B}\hat{C}\hat{D} \sim$ rectangle $ABCD$

$$\therefore \frac{\hat{A}\hat{B}}{AB} = \frac{\hat{B}\hat{C}}{BC} = \frac{\text{perimeter of rectangle } \hat{A}\hat{B}\hat{C}\hat{D}}{\text{perimeter of rectangle } ABCD}$$

= scale factor.

$$\therefore \frac{\hat{A}\hat{B}}{10} = \frac{\hat{B}\hat{C}}{6} = \frac{\text{perimeter of rectangle } \hat{A}\hat{B}\hat{C}\hat{D}}{32} = 3$$

$$\therefore \hat{A}\hat{B} = 30 \text{ cm.}, \hat{B}\hat{C} = 18 \text{ cm.}$$

, perimeter of rectangle $\hat{A}\hat{B}\hat{C}\hat{D} = 96 \text{ cm.}$

, area of rectangle $\hat{A}\hat{B}\hat{C}\hat{D} = 30 \times 18 = 540 \text{ cm}^2$

(The req.)

(2) Notice that the required rectangle is a shrinking
of the given rectangle and let rectangle $\hat{A}\hat{B}\hat{C}\hat{D}$
 $\sim$ rectangle $ABCD$

$$\therefore \frac{\hat{A}\hat{B}}{AB} = \frac{\hat{B}\hat{C}}{BC} = \frac{\text{perimeter of rectangle } \hat{A}\hat{B}\hat{C}\hat{D}}{\text{perimeter of rectangle } ABCD}$$

= scale factor.

$$\therefore \frac{\hat{A}\hat{B}}{10} = \frac{\hat{B}\hat{C}}{6} = \frac{\text{perimeter of rectangle } \hat{A}\hat{B}\hat{C}\hat{D}}{32} = 0.4$$

$$\therefore \hat{A}\hat{B} = 4 \text{ cm.}, \hat{B}\hat{C} = 2.4 \text{ cm.}$$

, perimeter of rectangle $\hat{A}\hat{B}\hat{C}\hat{D} = 12.8 \text{ cm.}$

, area of rectangle $\hat{A}\hat{B}\hat{C}\hat{D} = 4 \times 2.4 = 9.6 \text{ cm}^2$

(The req.)

Answers of Exercise 2

First Multiple choice questions

- | | | | |
|-------|--------|--------|--------|
| (1) b | (2) c | (3) a | (4) c |
| (5) b | (6) a | (7) d | (8) d |
| (9) c | (10) c | (11) a | (12) b |

Second Essay questions

1

(1) In $\triangle ABC$:

$$m(\angle A) = 180^\circ - (80^\circ + 55^\circ) = 45^\circ$$

$$\therefore m(\angle A) = m(\angle D) = 45^\circ$$

$$, m(\angle C) = m(\angle F) = 55^\circ$$

$$\therefore \triangle ABC \sim \triangle DEF$$

(2) In $\triangle ABC$:

$$m(\angle B) = 180^\circ - (65^\circ + 30^\circ) = 85^\circ$$

, in $\triangle XYZ$:

$$m(\angle X) = 180^\circ - (75^\circ + 65^\circ) = 40^\circ$$

$\therefore$ In $\triangle ABC$, XYZ :

$$\text{only } m(\angle A) = m(\angle Y)$$

$\therefore$ The two triangles are not similar.

(3) $\therefore \overline{AC} \parallel \overline{DB} \quad \therefore \triangle AEC \sim \triangle BED$

(4) $\therefore \triangle ABC, DEF$ are two equilateral triangles

$$\therefore \triangle ABC \sim \triangle DEF$$

(5) $\therefore \triangle ABC, XZY$ are isosceles triangles

$$, m(\angle B) = m(\angle Z) = 70^\circ$$

$$\therefore \triangle ABC \sim \triangle XZY$$

(6) $\therefore \frac{AD}{AB} \neq \frac{AE}{AC} \quad \therefore \triangle ADE, ABC$ aren't similar

(7) $\triangle XYZ \sim \triangle NLM$ because: $\frac{XY}{NL} = \frac{YZ}{LM} = \frac{XZ}{NM} = \frac{3}{2}$

(8) $\triangle AEC \sim \triangle BED$ because: $\frac{AE}{BE} = \frac{CE}{DE} = \frac{1}{2}$

$$, m(\angle AEC) = m(\angle BED) \text{ (V.O.A.)}$$

2

$$\therefore \frac{XB}{AB} = \frac{9}{12} = \frac{3}{4}, \frac{BY}{BC} = \frac{18}{24} = \frac{3}{4}, \frac{XY}{AC} = \frac{13.5}{18} = \frac{3}{4}$$

$$\therefore \frac{XB}{AB} = \frac{BY}{BC} = \frac{XY}{AC}$$

$$\therefore \triangle XBY \sim \triangle ABC$$

(Q.E.D. 1)

We deduce that:

$$m(\angle XBY) = m(\angle ABC)$$

$$\therefore \overline{BC} \text{ bisects } \angle ABX$$

(Q.E.D. 2)

3

$$\therefore \frac{AB}{DB} = \frac{6}{4} = \frac{3}{2}, \frac{BC}{BA} = \frac{9}{6} = \frac{3}{2}, \frac{AC}{DA} = \frac{7.5}{5} = \frac{3}{2}$$

$$\therefore \frac{AB}{DB} = \frac{BC}{BA} = \frac{AC}{DA}$$

$$\therefore \triangle ABC \sim \triangle DBA$$

(Q.E.D. 1)

We deduce that: $m(\angle ABD) = m(\angle ABC)$

$$\therefore \overline{BA} \text{ bisects } \angle DBC$$

(Q.E.D. 2)

SIMPLEST MATHS BOOKLETS

4

In ΔABM , ΔACB :

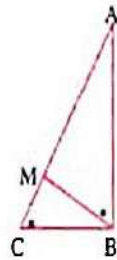
$\therefore \angle A$ is a common angle

$\therefore m(\angle ABM) = m(\angle C)$

$\therefore \Delta ABM \sim \Delta ACB$

$$\therefore \frac{AB}{AC} = \frac{AM}{AB}$$

$$\therefore (AB)^2 = AM \times AC \quad (\text{Q.E.D.})$$



5

(1) $\Delta ADE \sim \Delta ABC$

$\therefore \Delta ADX \sim \Delta ABY$, $\Delta AXE \sim \Delta AYC$

(2) $\therefore \Delta ADE \sim \Delta ABC$

$$\therefore \frac{AD}{AB} = \frac{DE}{BC} = \frac{AE}{AC} \quad (1)$$

$\therefore \Delta ADX \sim \Delta ABY$

$$\therefore \frac{AD}{AB} = \frac{DX}{BY} = \frac{AX}{AY} \quad (2)$$

$\therefore \Delta AXE \sim \Delta AYC$

$$\therefore \frac{AX}{AY} = \frac{XE}{YC} = \frac{AE}{AC} \quad (3)$$

from (1), (2), (3):

$$\therefore \frac{DX}{BY} = \frac{XE}{YC} = \frac{DE}{BC} \quad (\text{Q.E.D.})$$

6

$$\therefore \frac{AB}{DA} = \frac{CE}{BC} \quad \therefore \frac{AB}{CE} = \frac{DA}{BC}$$

$$\therefore \frac{BD}{DA} = \frac{EB}{BC} \quad \therefore \frac{BD}{EB} = \frac{DA}{BC}$$

$$\therefore \frac{AB}{CE} = \frac{DA}{BC} = \frac{BD}{EB}$$

$\therefore \Delta DBA \sim \Delta BEC$ We deduce that

$m(\angle ADB) = m(\angle CBE)$ and they are alternate angles

$$\therefore \overline{AD} \parallel \overline{BC} \quad (\text{Q.E.D. 1})$$

$m(\angle ABD) = m(\angle CEB)$ and they are alternate angles

$$\therefore \overline{AB} \parallel \overline{CE} \quad (\text{Q.E.D. 2})$$

7

$\therefore \angle A$, $\angle C$ subtended BD

$\therefore m(\angle A) = m(\angle C)$

$\therefore \angle E$ is a common angle

$\therefore \Delta ADE \sim \Delta CBE$

(First req.)

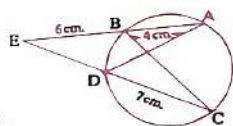
$$\therefore \frac{DE}{BE} = \frac{AE}{CE} \quad \therefore \frac{DE}{6} = \frac{10}{7+DE}$$

$$\therefore 7DE + (DE)^2 = 60$$

$$\therefore (DE)^2 + 7DE - 60 = 0$$

$$\therefore (DE + 12)(DE - 5) = 0 \quad \therefore DE = 5 \text{ cm.}$$

$$\therefore CE = 12 \text{ cm.} \quad (\text{Second req.})$$



8

$$\therefore DC = 2BD$$

$$\therefore AD = 6\sqrt{2} \text{ cm.}$$

$$\therefore (AD)^2 = DB \times DC$$

$$\therefore (6\sqrt{2})^2 = DB \times 2DB \quad \therefore 72 = 2(DB)^2$$

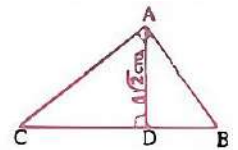
$$\therefore (DB)^2 = 36$$

$$\therefore BD = 6 \text{ cm.}$$

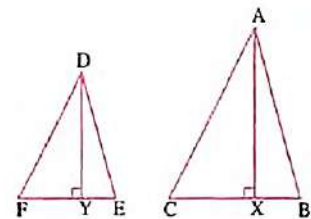
$$\therefore CD = 12 \text{ cm.} \quad \therefore (AB)^2 = 6 \times 18 = 108$$

$$\therefore AB = 6\sqrt{3} \text{ cm.}, (AC)^2 = 12 \times 18 = 216$$

$$\therefore AC = 6\sqrt{6} \text{ cm.} \quad (\text{The req.})$$



9



$\therefore \Delta ABC \sim \Delta DEF$

$\therefore m(\angle B) = m(\angle E)$, $m(\angle C) = m(\angle F)$

$\therefore$ In ΔABX , ΔDEY :

$\therefore m(\angle B) = m(\angle E)$

$\therefore m(\angle BXA) = m(\angle EYD) = 90^\circ$

$\therefore \Delta ABX \sim \Delta DEY$

$$\therefore \frac{BX}{EY} = \frac{AX}{DY} \quad (1)$$

In ΔAXC , ΔDYF :

$\therefore m(\angle C) = m(\angle F)$

$\therefore m(\angle AXC) = m(\angle DYF) = 90^\circ$

$\therefore \Delta AXC \sim \Delta DYF$

$$\therefore \frac{AX}{DY} = \frac{XC}{YF} \quad (2)$$

$$\text{From (1), (2):} \therefore \frac{BX}{EY} = \frac{XC}{YF}$$

$$\therefore BX \times YF = XC \times YE \quad (\text{Q.E.D.})$$

Answers of Exercise 3

First

Multiple choice questions

(1) d	(2) d	(3) a	(4) c
(5) c	(6) a	(7) d	(8) a
(9) c	(10) b	(11) c	(12) a
(13) a	(14) d	(15) d	(16) d
(17) a	(18) a	(19) c	(20) b

Second

Essay questions

1

$$\therefore \frac{\text{Area of the 1st triangle}}{\text{Area of the 2nd triangle}} = \left(\frac{3}{2}\right)^2 = \frac{9}{4}$$

Let area of the 1st triangle = $9x$

$\therefore$ Area of the 2nd triangle = $4x$

$$\therefore 9x + 4x = 130 \quad \therefore 13x = 130$$

$$\therefore x = 10$$

$\therefore$ Area of the 1st triangle = 90 cm^2

$\therefore$ area of the 2nd triangle = 40 cm^2 (The req.)

SIMPLEST MATHS BOOKLETS

2

∴ Ratio between lengths of two corresponding sides = 1 : 3

∴ Ratio between areas of the two polygons = 1 : 9

Let the area of the 1st polygon = x

∴ Area of the 2nd polygon = 9 x

∴ $9x - x = 32$ ∴ $8x = 32$ ∴ $x = 4$

∴ Area of the 1st polygon = 4 cm^2

∴ area of the 2nd polygon = 36 cm^2 (The req.)

3

∴ $DE \parallel BC$

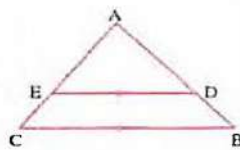
∴ $\triangle ADE \sim \triangle ABC$

$$\frac{\text{Area of } \triangle ADE}{\text{Area of } \triangle ABC} = \left(\frac{AD}{AB}\right)^2 = \left(\frac{2}{3}\right)^2$$

$$\frac{60}{\text{Area of } \triangle ABC} = \frac{4}{9}$$

∴ Area of $\triangle ABC = 135 \text{ cm}^2$

∴ Area of trapezium DBCE = $135 - 60 = 75 \text{ cm}^2$ (The req.)



4

∴ $FC \parallel AD$, DF is a transversal

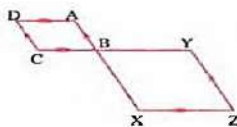
∴ $m(\angle F) = m(\angle ADE)$ (Alternate angles)

∴ $m(\angle C) = m(\angle A)$ (properties of a parallelogram)

∴ $\triangle DCF \sim \triangle EAD$

$$\frac{\text{Area of } (\triangle DCF)}{\text{Area of } (\triangle EAD)} = \left(\frac{DC}{EA}\right)^2 = \left(\frac{AB}{EA}\right)^2 = \frac{25}{9}$$

(The req.)



5

∴ $m(\angle ABC)$

$= m(\angle XBY)$ (V.O.A.)

∴ $m(\angle A) = m(\angle X)$

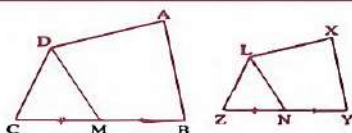
∴ $m(\angle C) = m(\angle Y)$

∴ $m(\angle D) = m(\angle Z)$

$$\frac{AB}{XB} = \frac{1}{2} \cdot \frac{BC}{BY} = \frac{1}{2} \quad \therefore \frac{CD}{YZ} = \frac{1}{2}, \frac{AD}{XZ} = \frac{1}{2}$$

∴ Parallelogram ABCD $\sim$ parallelogram XBYZ

$$\frac{\text{a (parallelogram ABCD)}}{\text{a (parallelogram XBYZ)}} = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \quad (\text{Q.E.D.})$$



6

∴ The two polygons are similar.

∴ $\triangle MDC \sim \triangle NLZ$ ∴ $\frac{MD}{NL} = \frac{DC}{LZ}$

$$\frac{\text{a (polygon ABCD)}}{\text{a (polygon XYZL)}} = \left(\frac{DC}{LZ}\right)^2 = \left(\frac{MD}{NL}\right)^2$$

$$\therefore \text{a (polygon ABCD)} : \text{a (polygon XYZL)} = (MD)^2 : (NL)^2 \quad (\text{Q.E.D.})$$

7

∴ $\triangle ABX$, $\triangle BCY$, $\triangle ACZ$

are equilateral triangles

∴ $\triangle ABX \sim \triangle BCY \sim \triangle ACZ$

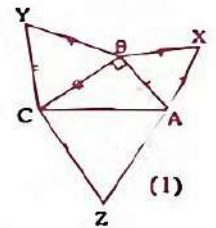
$$\therefore \frac{\text{a}(\triangle ABX)}{\text{a}(\triangle ACZ)} = \left(\frac{AB}{AC}\right)^2 = \frac{(AB)^2}{(AC)^2}$$

$$\therefore \frac{\text{a}(\triangle BCY)}{\text{a}(\triangle ACZ)} = \left(\frac{BC}{AC}\right)^2 = \frac{(BC)^2}{(AC)^2} \quad (2)$$

Adding (1) + (2) :

$$\therefore \frac{\text{a}(\triangle ABX) + \text{a}(\triangle BCY)}{\text{a}(\triangle ACZ)} = \frac{(AB)^2 + (BC)^2}{(AC)^2} = \frac{(AC)^2}{(AC)^2}$$

$$\therefore \text{a}(\triangle ABX) + \text{a}(\triangle BCY) = \text{a}(\triangle ACZ) \quad (\text{Q.E.D.})$$



8

In $\triangle BCE$, $\triangle ABE$:

∴ $m(\angle CBE)$ (tangency)

$= m(\angle A)$ (inscribed)

∴ $\angle E$ is common

∴ $\triangle BCE \sim \triangle ABE$

$$\therefore \frac{\text{a}(\triangle BCE)}{\text{a}(\triangle ABE)} = \left(\frac{BC}{AB}\right)^2 = \frac{9}{16}$$

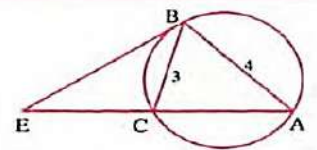
∴ $\text{a}(\triangle BCE) = 9x$

∴ $\text{a}(\triangle ABE) = 16x$

∴ $\text{a}(\triangle ABC) = \text{a}(\triangle ABE) - \text{a}(\triangle BCE)$

$$= 16x - 9x = 7x$$

$$\therefore \frac{\text{a}(\triangle ABC)}{\text{a}(\triangle ABE)} = \frac{7x}{16x} = \frac{7}{16} \quad (\text{Q.E.D.})$$



9

∴ $\triangle ABD \sim \triangle BCD$

$$\therefore \frac{AB}{BC} = \frac{BD}{CD} = \frac{AD}{BD}$$

$$\therefore \frac{AB}{BC} = \frac{AX}{BM} = \frac{XY}{MN} = \frac{BY}{CN}$$

$$\therefore \frac{BD}{CD} = \frac{AD}{BD} = \frac{AB}{BC} = \frac{AX}{BM} = \frac{XY}{MN} = \frac{BY}{CN}$$

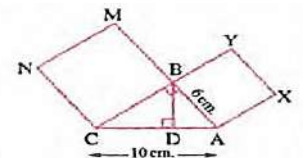
∴ Lengths of corresponding sides in the two polygons DAXYB, DBMNC are proportional.

∴ measures of corresponding angles in the two polygons DAXYB, DBMNC are equal (Why) ?

∴ Polygon DAXYB $\sim$ Polygon DBMNC (First req.)

$$\therefore BC = \sqrt{100 - 36} = 8 \text{ cm.}$$

$$\therefore \frac{\text{a (polygon DAXYB)}}{\text{a (polygon DBMNC)}} = \left(\frac{6}{8}\right)^2 = \frac{36}{64} = \frac{9}{16} \quad (\text{Second req.})$$



SIMPLEST MATHS BOOKLETS

Answers of Exercise 4

First Multiple choice questions

- | | | | |
|--------|--------|--------|--------|
| (1) b | (2) d | (3) a | (4) c |
| (5) a | (6) a | (7) d | (8) b |
| (9) a | (10) c | (11) c | (12) a |
| (13) c | (14) b | (15) a | (16) a |
| (17) c | (18) b | (19) a | (20) c |

Second Essay questions

1

- (1) $\therefore AE \times EB = 6 \times 7 = 42$
 $\therefore CE \times ED = 5 \times 8.4 = 42$
 $\therefore AE \times EB = CE \times ED$
 $\therefore$ The points A, B, C, D lie on one circle.
- (2), (3) The points A, B, C, D are not lie on one circle because points A, B, D lie on one straight line.
- (4) $\therefore AE \times EB = 5 \times 20 = 100$
 $\therefore CE \times ED = 10 \times 10 = 100$
 $\therefore AE \times EB = CE \times ED$
 $\therefore$ The points A, B, C, D lie on same circle.
- (5) $\therefore AE \times BE = 12 \times 3 = 36$
 $\therefore CE \times DE = 9 \times 4 = 36$
 $\therefore$ The points A, B, C, D lie on one circle.
- (6) $\therefore AE \times BE = 6 \times 3.6 = 21.6$
 $\therefore CE \times DE = 7.2 \times 2.8 = 20.16$
 $\therefore AE \times BE \neq CE \times DE$
 $\therefore$ The points A, B, C, D are not lie on one circle.

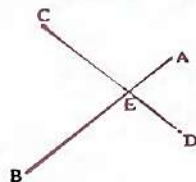
2 (1), (4), (6)

3

- $\therefore \triangle XLM, \triangle XZY$ have :
 $\frac{XL}{XZ} = \frac{4}{8} = \frac{1}{2}, \frac{XM}{XY} = \frac{6}{12} = \frac{1}{2}$
 $\therefore \angle X$ is a common angle.
 $\therefore \triangle XLM \sim \triangle XZY$ (Q.E.D. 1)
 $\therefore \frac{XL}{XZ} = \frac{XM}{XY} \therefore XL \times XY = XM \times XZ$
 $\therefore$ Figure LYZM is a cyclic quadrilateral (Q.E.D. 2)

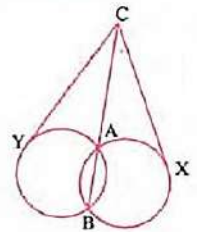
4

- $\therefore AE = \frac{5}{12} BE, BE = 6 \text{ cm.}$
 $\therefore AE = 2.5 \text{ cm.}$
 $\therefore DE = \frac{3}{5} CE, EC = 5 \text{ cm.}$
 $\therefore DE = 3 \text{ cm.}$
 $\therefore AE \times BE = 2.5 \times 6 = 15, DE \times EC = 3 \times 5 = 15$
 $\therefore AE \times BE = DE \times EC$
 $\therefore$ The points A, B, C, D lie on the same circle. (Q.E.D.)



5

$$\begin{aligned} \therefore (CX)^2 &= CA \times CB \\ \therefore (CY)^2 &= CA \times CB \\ \therefore CX &= CY \quad (\text{Q.E.D.}) \end{aligned}$$

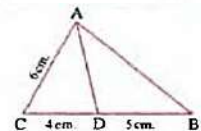


6

- In circle M :
 $\therefore (AB)^2 = AF \times AE \therefore (AB)^2 = 4 \times 9 = 36$
 $\therefore AB = 6 \text{ cm.}$ (1)
- In circle N : $(AC)^2 = AE \times AD = 9 \times 16 = 144$
 $\therefore AC = 12 \text{ cm.}$ (2)
- From (1), (2) : $\therefore AB = \frac{1}{2} AC$
 $\therefore B$ is the midpoint of $\overline{AC}$ (Q.E.D.)

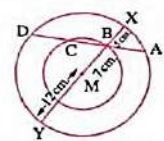
7

- $\therefore (AC)^2 = CD \times BC$
 $\therefore \overline{AC}$ is a tangent to the circle passing through the points A, B, D (Q.E.D. 1)
- $\therefore \triangle ACD, \triangle BCA$ have :
 $m(\angle DAC) = m(\angle B)$ (tangency and inscribed angles subtended by $\widehat{AD}$)
 $\therefore \angle C$ is a common angle.
 $\therefore \triangle ACD \sim \triangle BCA$ (Q.E.D. 2)
- $\therefore \frac{a(\triangle ACD)}{a(\triangle BCA)} = \left(\frac{CD}{CA}\right)^2 = \left(\frac{4}{6}\right)^2 = \frac{4}{9}$
 $\therefore a(\triangle ACD) = 4k, a(\triangle BCA) = 9k$
 $\therefore a(\triangle ABD) = 5k$
 $\therefore \frac{a(\triangle ABD)}{a(\triangle ABC)} = \frac{5k}{9k} = \frac{5}{9}$ (Q.E.D. 3)



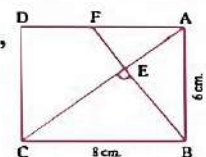
8 Construction :

- Draw the diameter $\overline{XY}$ in the major circle, intersecting the minor circle at B
Proof : $\therefore \overline{AD} \cap \overline{XY} = \{B\}$
 $\therefore AB \times BD = XB \times BY = 5 \times 19 = 95$ (Q.E.D.)



9

- $\therefore \triangle ABC$ is right-angled at B, $\overline{BE} \perp \overline{AC}$
 $\therefore (AB)^2 = AE \times AC$ (1)
 $\therefore$ Figure FECD is a cyclic quadrilateral. (because $m(\angle D) + m(\angle FEC) = 180^\circ$)
 $\therefore AF \times AD = AE \times AC$ (2)
- From (1), (2) :
 $\therefore (AB)^2 = AF \times AD$ (First req.)
 $\therefore (6)^2 = AF \times 8$
 $\therefore AF = 4.5 \text{ cm.}$ (Second req.)



SIMPLEST MATHS BOOKLETS

Answers of Exercise 5

First Multiple choice questions

- (1) First : b Second : d Third : b (2) d
(3) c (4) b (5) b (6) b
(7) c (8) a

Second Essay questions

1

$$(1) \because \frac{AD}{DB} = \frac{15}{9} = \frac{5}{3}, \frac{AE}{EC} = \frac{18}{12} = \frac{3}{2}$$

$$\therefore \frac{AD}{DB} \neq \frac{AE}{EC} \therefore \overline{DE} \text{ is not parallel to } \overline{BC}$$

$$(2) \because \frac{CA}{AE} = \frac{45}{63} = \frac{5}{7}, \frac{BA}{AD} = \frac{55}{77} = \frac{5}{7}$$

$$\therefore \frac{CA}{AE} = \frac{BA}{AD} \therefore \overline{DE} \parallel \overline{BC}$$

$$(3) \because \frac{DA}{AB} = \frac{3}{4}, \frac{EA}{AC} = \frac{6}{8} = \frac{3}{4}$$

$$\therefore \frac{DA}{AB} = \frac{EA}{AC} \therefore \overline{DE} \parallel \overline{BC}$$

$$(4) \because \frac{AD}{DB} = \frac{6}{10} = \frac{3}{5}, \frac{AE}{EC} = \frac{9}{15} = \frac{3}{5}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \therefore \overline{DE} \parallel \overline{BC}$$

$$(5) \because \frac{AD}{DB} = \frac{28}{20} = \frac{7}{5}, \frac{AE}{EC} = \frac{42}{24} = \frac{7}{4}$$

$$\therefore \frac{AD}{DB} \neq \frac{AE}{EC} \therefore \overline{DE} \text{ is not parallel to } \overline{BC}$$

(6) In the right-angled triangle AED at E :

$$(AD)^2 = (AE)^2 + (ED)^2 = 225 + 400 = 625$$

$$\therefore AD = 25 \text{ cm.}$$

$$\because \frac{AE}{EC} = \frac{15}{9} = \frac{5}{3}, \frac{AD}{DB} = \frac{25}{15} = \frac{5}{3}$$

$$\therefore \frac{AE}{EC} = \frac{AD}{DB} \therefore \overline{DE} \parallel \overline{BC}$$

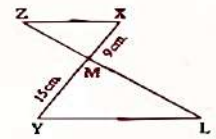
2

$$\because \overline{XZ} \parallel \overline{LY}$$

$$\therefore \frac{ZM}{ML} = \frac{XM}{MY}$$

$$\therefore \frac{ZM}{36} = \frac{9}{24}$$

$$\therefore ZM = 13.5 \text{ cm.}$$



(The req.)

3

$$(1) \because \overline{DE} \parallel \overline{BC}$$

$$\therefore \frac{4}{8} = \frac{x}{6}$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC}$$

$$\therefore x = 3$$

$$(2) \because \overline{DE} \parallel \overline{BC}$$

$$\therefore \frac{x}{5} = \frac{x-2}{3}$$

$$\therefore 2x = 10$$

$$\therefore \frac{AE}{EC} = \frac{AD}{DB}$$

$$\therefore 5x - 10 = 3x$$

$$\therefore x = 5$$

$$(3) \because \overline{DF} \parallel \overline{AC}$$

$$\therefore \frac{x}{21} = \frac{6}{14}$$

$$\therefore \frac{AD}{AB} = \frac{CF}{CB}$$

$$\therefore x = 9$$

$$(4) \because 2DB = 12$$

$$\therefore 3FC = 12$$

$$\therefore \overline{DF} \parallel \overline{AC}$$

$$\therefore \frac{x}{6} = \frac{4}{x+5}$$

$$\therefore x^2 + 5x - 24 = 0$$

$$\therefore x = -8 \text{ (refused) or } x = 3$$

$$\therefore DB = 6 \text{ cm.}$$

$$\therefore FC = 4 \text{ cm.}$$

$$\therefore \frac{AD}{DB} = \frac{CF}{FB}$$

$$\therefore x(x+5) = 24$$

$$\therefore (x+8)(x-3) = 0$$

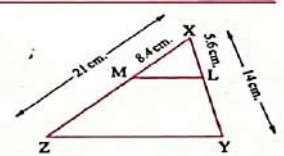
4

$$\therefore \frac{XL}{XY} = \frac{5.6}{14} = \frac{2}{5}$$

$$\therefore \frac{XM}{XZ} = \frac{8.4}{21} = \frac{2}{5}$$

$$\therefore \frac{XL}{XY} = \frac{XM}{XZ}$$

$$\therefore \overline{LM} \parallel \overline{YZ}$$



(Q.E.D.)

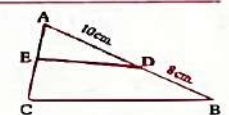
5

$$\therefore 5AE = 4EC$$

$$\therefore \frac{AE}{EC} = \frac{4}{5}$$

$$\therefore \frac{AD}{DB} = \frac{10}{8} = \frac{5}{4}$$

$$\therefore \overline{DE} \text{ is not parallel to } \overline{BC}$$



$$\therefore \frac{AE}{EC} \neq \frac{AD}{DB}$$

(Q.E.D.)

6

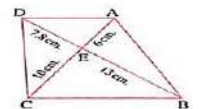
$$\therefore \frac{AE}{EC} = \frac{6}{10} = \frac{3}{5}$$

$$\therefore \frac{DE}{EB} = \frac{7.8}{13} = \frac{3}{5}$$

$$\therefore \frac{AE}{EC} = \frac{DE}{EB}$$

$$\therefore \overline{AD} \parallel \overline{BC}$$

$$\therefore ABCD \text{ is a trapezium.}$$



(Q.E.D.)

7

Given : $\triangle ABC$, D is the midpoint of AB, E is the midpoint of AC

R.T.P. : (1) $\overline{DE} \parallel \overline{BC}$

$$(2) DE = \frac{1}{2} BC$$

Construction : Draw $\overline{EF} \parallel \overline{AB}$ to intersect $\overline{BC}$ at F

$$\text{Proof : } \because \frac{AD}{DB} = 1, \frac{AE}{EC} = 1$$

$$\therefore \frac{AD}{DB} = \frac{AE}{EC} \therefore \overline{DE} \parallel \overline{BC}$$

(Q.E.D. 1)

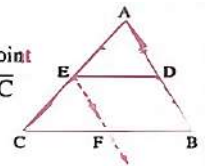
$\because \overline{EF} \parallel \overline{AB}$, E is the midpoint of $\overline{AC}$

$\therefore$ F is the midpoint of $\overline{BC}$ $\therefore BF = \frac{1}{2} BC$

$\therefore$ the figure BDEF is a parallelogram.

$$\therefore DE = BF = \frac{1}{2} BC$$

(Q.E.D. 2)



SIMPLEST MATHS BOOKLETS

8

In $\triangle ABC$: $\therefore 3AD = 2DB$

$$\therefore \frac{AD}{DB} = \frac{2}{3}$$

$$\therefore \frac{AF}{FX} = \frac{8}{12} = \frac{2}{3}$$

$$\therefore \frac{AD}{DB} = \frac{AF}{FX}$$

$$\therefore \overline{DF} \parallel \overline{BX} \quad (1)$$

In $\triangle AXC$: $\therefore 5CE = 3AC$ $\therefore \frac{AC}{CE} = \frac{5}{3}$

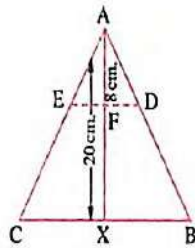
$$\therefore \frac{AX}{XF} = \frac{20}{12} = \frac{5}{3}$$

$$\therefore \frac{AC}{CE} = \frac{AX}{XF}$$

$$\therefore \overline{FE} \parallel \overline{XC} \quad (2)$$

From (1), (2) : $\therefore \overline{DE} \parallel \overline{BC}$

$\therefore$ The points D, F and E are collinear (Q.E.D.)



9

In $\triangle ADY$: $\therefore \overline{EX} \parallel \overline{DY}$

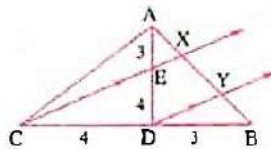
$$\therefore \frac{AE}{ED} = \frac{AX}{XY} = \frac{3}{4} \quad (1)$$

In $\triangle BCX$: $\therefore \overline{DY} \parallel \overline{CX}$

$$\therefore \frac{BD}{DC} = \frac{BY}{XY} = \frac{3}{4} \quad (2)$$

From (1), (2) : $\therefore \frac{AX}{XY} = \frac{BY}{XY}$

$$\therefore AX = BY \quad (Q.E.D.)$$



Answers of Exercise 6

First Multiple choice questions

- (1) b (2) d (3) c (4) b
(5) b (6) c (7) b (8) b
(9) b (10) c

Second Essay questions

- 1
(1) EF (2) DF (3) DE (4) DF
(5) ME (6) DF (7) ME (8) MC

- 2
(1) $\therefore \overline{AB} \parallel \overline{DE}$, $BE = EC$ $\therefore AD = DC$
 $\therefore 3x - 1 = 2x + 3$ $\therefore x = 4$
 $\therefore BE = EC$ $\therefore 2y + 7 = 13$
 $\therefore 2y = 6$ $\therefore y = 3$
(2) $\therefore \overline{AD} \parallel \overline{BE} \parallel \overline{CF}$, $DE = EF = FM$
 $\therefore AB = BC = CM$ $\therefore x^2 - 3 = 3x + 1$
 $\therefore x^2 - 3x - 4 = 0$ $\therefore (x - 4)(x + 1) = 0$
 $\therefore x = 4$ or $x = -1$ (refused)
 $\therefore BC = CM$ $\therefore 2y - 1 = 13$
 $\therefore 2y = 14$ $\therefore y = 7$
(3) $\therefore \overline{AB} \parallel \overline{DC} \parallel \overline{EF}$ $\therefore \frac{AM}{BM} = \frac{MD}{MC} = \frac{DF}{CE}$
 $\therefore \frac{y - 4}{4x - 1} = \frac{2}{3} = \frac{y - 4}{2x + 7}$ $\therefore 2x = 8$
 $\therefore 4x - 1 = 2x + 7$ $\therefore \frac{y - 4}{15} = \frac{2}{3}$
 $\therefore x = 4$ $\therefore y = 14$
 $\therefore y - 4 = 10$
(4) $\therefore \overline{AB} \parallel \overline{CD} \parallel \overline{EF}$, $BD = DF$
 $\therefore AC = CE$ $\therefore 2x - 3 = x + 2$
 $\therefore x = 5$
 $\therefore BD = DF$ $\therefore y + 3 = 6$ $\therefore y = 3$

$$(5) \therefore x + 3 = 2y + 5$$

$$\therefore x - 2y = 2 \quad (1)$$

$$\therefore x - 3 = y + 2 \quad \therefore x - y = 5 \quad (2)$$

by subtracting (1) from (2) :

$$\therefore y = 3, \text{ by substituting in (2) :}$$

$$\therefore x = 8$$

$$(6) \therefore \overline{DE} \parallel \overline{BC}, AD = DB \quad \therefore AE = EC$$

$$\therefore x + 6 = 3x - 2 \quad \therefore 2x = 8 \quad \therefore x = 4$$

In $\triangle ABC$:

$\therefore D, E$ are the midpoints of $\overline{AB}, \overline{AC}$ respectively

$$\therefore DE = \frac{1}{2} BC \quad \therefore 3y - 2 = \frac{1}{2}(5y - 1)$$

$$\therefore 6y - 4 = 5y - 1 \quad \therefore y = 3$$

$$(7) \therefore 2x + 1 = y - 3 \quad \therefore x^2 - 5 = 3x - 1$$

$$\therefore x^2 - 3x - 4 = 0 \quad \therefore (x - 4)(x + 1) = 0$$

$$\therefore x = 4 \text{ or } x = -1 \text{ (refused)}$$

$$\therefore y - 3 = 9 \quad \therefore y = 12$$

$$(8) \therefore \frac{3x + 2}{15} = \frac{2x + 4}{12}$$

$$\therefore 12(3x + 2) = 15(2x + 4)$$

$$\therefore 36x + 24 = 30x + 60 \quad \therefore 6x = 36$$

$$\therefore x = 6 \quad \therefore \frac{y}{15} = \frac{11}{12} \quad \therefore y = 13\frac{3}{4}$$

$$(9) \therefore \frac{x + 1}{9} = \frac{6}{5} = \frac{10}{y}$$

$$\therefore x + 1 = \frac{54}{5} = 10\frac{4}{5} \quad \therefore x = 9\frac{4}{5}$$

$$\therefore y = \frac{50}{6} = 8\frac{1}{3}$$

3

$$\therefore \overline{AC} \parallel \overline{FE} \parallel \overline{DB}$$

$$\therefore \frac{ME}{MF} = \frac{EB}{FD}$$

$$\therefore \frac{6}{MF} = \frac{9}{15}$$

$$\therefore MF = 10 \text{ cm. (First req.)}$$

$$\therefore \frac{AM}{MB} = \frac{CM}{MD}$$

$$\therefore \frac{AM}{15} = \frac{18}{25}$$

$$\therefore AM = 10.8 \text{ cm.}$$

(Second req.)

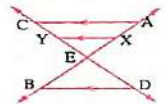
4

$$\therefore \overline{XY} \parallel \overline{BD} \parallel \overline{AC}$$

$$\therefore \frac{AX}{CY} = \frac{EB}{ED}$$

$$\therefore AX \times ED = CY \times EB$$

(Q.E.D.)



5

In $\triangle ABC$:

$\therefore E$ is the midpoint of $\overline{BC}$

$\therefore \overline{EY} \parallel \overline{AB}$

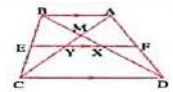
$\therefore Y$ is the midpoint of $\overline{AC}$:

$$EY = \frac{1}{2} AB \quad (Q.E.D. 1)$$

$\therefore \overline{AB} \parallel \overline{EE} \parallel \overline{DC}$ and $\overline{AC}, \overline{DB}$ are two transversals

$$\therefore \frac{AM}{BM} = \frac{MY}{MX} = \frac{YC}{XD} \quad \therefore \frac{AM + MY}{BM + MX} = \frac{MY + YC}{MX + XD}$$

$$\therefore \frac{AY}{BX} = \frac{MC}{MD} \quad \therefore \frac{AY}{MC} = \frac{BX}{DM} \quad (Q.E.D. 2)$$



6

It is possible to find $\frac{AB}{BC}$ by three methods :

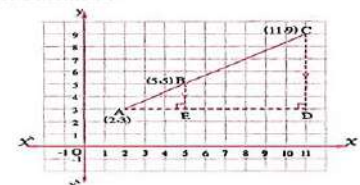
First method : Using the distance between two points in the cartesian plane :

$$\therefore AB = \sqrt{(5-2)^2 + (5-3)^2} = \sqrt{13} \text{ length unit.}$$

$$\therefore BC = \sqrt{(11-5)^2 + (9-5)^2} = 2\sqrt{13} \text{ length unit.}$$

$$\therefore \frac{AB}{BC} = \frac{\sqrt{13}}{2\sqrt{13}} = \frac{1}{2}$$

Second method :



SIMPLEST MATHS BOOKLETS

Answers of Exercise 7

First Multiple choice questions

- (1) d (2) c (3) a (4) b
 (5) c (6) c (7) a (8) c
 (9) d (10) a (11) c (12) c
 (13) d (14) c (15) b (16) d
 (17) a (18) b (19) b (20) c

Second Essay questions

1

- (1) $\because \overline{BD}$ bisects $\angle ABC$ $\therefore \frac{CD}{DA} = \frac{CB}{BA}$
 $\therefore \frac{X+1}{5} = \frac{X+4}{8}$ $\therefore 8X+8 = 5X+20$
 $\therefore 3X = 12$ $\therefore X = 4$
 (2) $\because \overline{AD}$ bisects $\angle BAC$ $\therefore \frac{BD}{DC} = \frac{BA}{AC}$
 $\therefore \frac{6X}{5X} = \frac{10X+4}{9X+2}$
 $\therefore 6X(9X+2) = 5X(10X+4)$
 $\therefore 54X^2 + 12X = 50X^2 + 20X$
 $\therefore 4X^2 - 8X = 0$ $\therefore 4X(X-2) = 0$
 $\therefore X = 0$ (refused) or $X = 2$

2

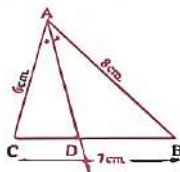
- (1) $\because m(\angle B) = m(\angle C)$ $\therefore AB = AC = 7$ cm.
 $\because \overline{AD}$ bisects $\angle BAC$ $\therefore \frac{BD}{DC} = \frac{AB}{AC} = 1$
 $\therefore \frac{X}{4} = 1$ $\therefore X = 4$
 $\therefore$ The perimeter of $\triangle ABC = 7 + 7 + 8 = 22$ cm.
 (2) In $\triangle ADC$ which is right-angled at D
 $(DC)^2 = (50)^2 - (30)^2 = 1600$
 $\therefore DC = 40$ cm.

- $\because \overline{AB}$ bisects $\angle DAC$
 $\therefore \frac{DB}{BC} = \frac{DA}{AC} = \frac{30}{50} = \frac{3}{5}$ $\therefore \frac{DB+BC}{BC} = \frac{3+5}{5}$
 $\therefore \frac{DC}{X} = \frac{8}{5}$ $\therefore \frac{40}{X} = \frac{8}{5}$
 $\therefore X = 25$ $\therefore DB = 40 - 25 = 15$ cm.
 $\therefore AB = \sqrt{DA \times AC - BD \times BC}$
 $= \sqrt{50 \times 30 - 15 \times 25} = 15\sqrt{5}$ cm.
 $\therefore$ The perimeter of $\triangle ABC = 15\sqrt{5} + 50 + 25$
 $= (75 + 15\sqrt{5})$ cm.

- 3) $\because \overline{BD}$ bisects $\angle ABC$ $\therefore \frac{AD}{DC} = \frac{AB}{BC}$
 $\therefore \frac{4}{X} = \frac{6}{X+3}$ $\therefore 6X = 4X + 12$
 $\therefore 2X = 12$ $\therefore X = 6$
 $\therefore$ The perimeter of $\triangle ABC = 6 + 9 + 10 = 25$ cm.

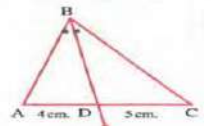
3

- $\because \overline{AD}$ bisects $\angle BAC$
 $\therefore \frac{BD}{DC} = \frac{BA}{AC} = \frac{8}{6} = \frac{4}{3}$
 $\therefore \frac{BD+DC}{DC} = \frac{4+3}{3}$
 $\therefore \frac{BC}{DC} = \frac{7}{3}$ $\therefore \frac{7}{DC} = \frac{7}{3}$ $\therefore DC = 3$ cm.
 $\therefore BD = 7 - 3 = 4$ cm. (The req.)



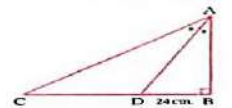
4

- $\because \overline{BD}$ bisects $\angle B$
 $\therefore \frac{AB}{BC} = \frac{AD}{DC} = \frac{4}{5}$
 $\therefore \frac{AB+BC}{BC} = \frac{4+5}{5}$
 $\therefore$ the perimeter of the triangle = 27 cm, $\therefore AC = 9$ cm.
 $\therefore AB + BC = 27 - 9 = 18$ cm.
 $\therefore \frac{18}{5} = \frac{9}{5}$ $\therefore BC = 10$ cm. $\therefore AB = 8$ cm.
 $\therefore BD = \sqrt{AB \times BC - AD \times DC} = \sqrt{10 \times 8 - 4 \times 5}$
 $= 2\sqrt{15}$ cm. (The req.)



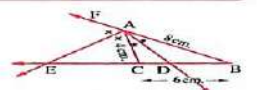
5

- $\because \overline{AD}$ bisects $\angle BAC$
 $\therefore \frac{BD}{DC} = \frac{BA}{AC} = \frac{3}{5}$
 $\therefore \frac{24}{DC} = \frac{3}{5}$
 $\therefore DC = 40$ cm. $\therefore BC = 40 + 24 = 64$ cm.
 $\therefore \frac{AB}{AC} = \frac{3}{5}$ $\therefore AB = 3$ m, $AC = 5$ m.
 From pythagoras' theorem $\therefore BC = 4$ m.
 $\therefore 4$ m = 64 $\therefore m = 16$
 by substituting $\therefore AB = 3 \times 16 = 48$ cm.
 $\therefore AC = 5 \times 16 = 80$ cm.
 $\therefore$ The perimeter of $\triangle ABC = 80 + 48 + 64 = 192$ cm.
 (The req.)



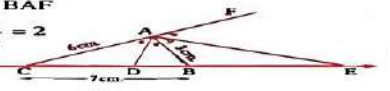
6

- $\because \overline{AE}$ bisects $\angle CAF$
 $\therefore \frac{BE}{CE} = \frac{BA}{AC}$
 $\therefore \frac{CE+6}{CE} = \frac{8}{4} = \frac{2}{1}$
 $\therefore 2CE = CE + 6$ $\therefore CE = 6$ cm.
 $\because \overline{AD}$ bisects $\angle BAC$ $\therefore \frac{BA}{AC} = \frac{BD}{DC}$
 $\therefore \frac{BD}{DC} = \frac{8}{4} = \frac{2}{1}$ $\therefore \frac{BD+DC}{DC} = \frac{8+4}{4}$
 $\therefore \frac{BC}{DC} = \frac{12}{4}$ $\therefore \frac{6}{DC} = 3$
 $\therefore DC = 2$ cm.
 $\therefore DE = CE + CD = 6 + 2 = 8$ cm.
 $\therefore BD = 6 - 2 = 4$ cm.
 $\therefore AD = \sqrt{BA \times AC - BD \times DC} = \sqrt{8 \times 4 - 4 \times 2}$
 $= 2\sqrt{6}$ cm.
 $\therefore AE = \sqrt{BE \times CE - BA \times AC} = \sqrt{12 \times 6 - 8 \times 4}$
 $= 2\sqrt{10}$ cm. (The req.)



7

- $\because \overline{AE}$ bisects $\angle BAF$
 $\therefore \frac{CE}{EB} = \frac{AC}{AB} = \frac{6}{3} = 2$
 $\therefore CE = 2EB$ $\therefore CB = BE$
 $\therefore B$ is the midpoint of $\overline{CE}$
 $\therefore \overline{AB}$ is a median of $\triangle ACE$ (First req.)
 $\because \overline{AD}$ bisects $\angle BAC$ $\therefore \frac{BD}{DC} = \frac{BA}{AC} = \frac{3}{6} = \frac{1}{2}$
 $\therefore BD = \frac{7}{3}$ cm, $DC = \frac{14}{3}$ cm, $BE = 7$ cm.
 $\therefore ED = \frac{7}{3} + 7 = \frac{28}{3}$ cm, $EC = 14$ cm.
 $\therefore \frac{\text{The area of } (\triangle ADE)}{\text{The area of } (\triangle ACE)} = \frac{ED}{CE} = \frac{28}{14} = \frac{2}{3}$
 (because they have the same height) (Second req.)



8

- (1) $\because \overline{CE}$ bisects $\angle ACB$
 $\therefore \frac{BE}{AE} = \frac{BC}{CA} = \frac{6}{12} = \frac{1}{2}$
 $\therefore \frac{CP}{FA} = \frac{4}{8} = \frac{1}{2}$ $\therefore \frac{BE}{AE} = \frac{CP}{FA}$
 $\therefore \overline{EE} \parallel \overline{BC}$ (Q.E.D.)
 (2) In $\triangle ABD$: $\because \overline{BE}$ bisects $\angle ABD$
 $\therefore \frac{AE}{ED} = \frac{AB}{BD}$ (1)
 In $\triangle ADC$: $\because \overline{DF}$ bisects $\angle ADC$
 $\therefore \frac{AF}{FC} = \frac{AD}{DC}$ (2)
 $\therefore AB = AD$, $BD = DC$ (3)
 From (1), (2), (3) : $\therefore \frac{AE}{ED} = \frac{AF}{FC}$
 In $\triangle ADC$: $\therefore \overline{EE} \parallel \overline{DC}$ $\therefore \overline{EF} \parallel \overline{BC}$ (Q.E.D.)

SIMPLEST MATHS BOOKLETS

Answers of Exercise 8

First Multiple choice questions

- (1) a (2) d (3) c (4) c
(5) d (6) b (7) d (8) b

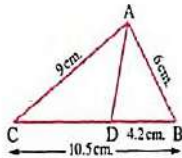
Second Essay questions

1

$$\therefore \frac{AB}{AC} = \frac{6}{9} = \frac{2}{3}$$

$$\frac{BD}{DC} = \frac{4.2}{10.5 - 4.2} = \frac{2}{3}$$

$$\therefore \frac{AB}{AC} = \frac{BD}{DC} \therefore \overline{AD} \text{ bisects } \angle BAC \quad (\text{Q.E.D.})$$

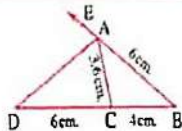


2

$$\therefore \frac{BA}{AC} = \frac{6}{3.6} = \frac{5}{3}$$

$$\frac{BD}{DC} = \frac{4 + 6}{6} = \frac{5}{3}$$

$$\therefore \frac{BA}{AC} = \frac{BD}{DC} \therefore \overline{AD} \text{ bisects } \angle CAE \quad (\text{Q.E.D.})$$



3

(1) In $\triangle ADC$: $\therefore \overline{DE}$ bisects $\angle ADC$

$$\therefore \frac{CE}{EA} = \frac{CD}{DA} = \frac{28}{42} = \frac{2}{3}$$

$$\therefore \frac{BC}{BA} = \frac{36}{54} = \frac{2}{3}$$

$$\therefore \frac{CE}{EA} = \frac{BC}{BA}$$

$\therefore \overline{BE}$ bisects $\angle ABC$ in $\triangle ABC$ (Q.E.D.)

(2) $\therefore \triangle ABC$ is right angled at A

$$\therefore (BC)^2 = (AB)^2 + (AC)^2 = (30)^2 + (40)^2 = 2500$$

$$\therefore BC = 50 \text{ cm.}$$

$$\therefore \overline{AD} \perp \overline{BC}$$

$$\therefore AD = \frac{BA \times AC}{BC} \quad (\text{Euclid theorem})$$

$$\therefore AD = 24 \text{ cm.}$$

$$\therefore AE = 24 - 9 = 15 \text{ cm.}$$

$$\therefore \triangle ABC \sim \triangle DBA$$

$$\therefore \frac{AB}{DB} = \frac{BC}{AB}$$

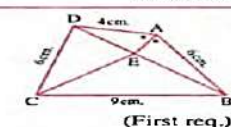
$$\therefore \frac{30}{DB} = \frac{50}{30}$$

$$\therefore DB = 18 \text{ cm.}$$

$$\therefore \frac{DB}{BA} = \frac{18}{30} = \frac{3}{5}, \quad \frac{DE}{EA} = \frac{9}{15} = \frac{3}{5}$$

$$\therefore \frac{DB}{BA} = \frac{DE}{EA}$$

$\therefore \overline{BE}$ bisects $\angle ABC$ (Q.E.D.)



4

$\therefore \overline{AE}$ bisects $\angle DAB$

$$\therefore \frac{AB}{AD} = \frac{BE}{ED}$$

$$\therefore \frac{BE}{ED} = \frac{6}{4} = \frac{3}{2}$$

$$\therefore \frac{BC}{CD} = \frac{9}{6} = \frac{3}{2}$$

$\therefore \overline{CE}$ bisects $\angle BCD$ (Second req.)

$$\therefore \frac{BE}{ED} = \frac{CB}{CD}$$

5

In $\triangle ACD$: $\therefore 2AE = 3ED$

$$\therefore \frac{AE}{ED} = \frac{3}{2}$$

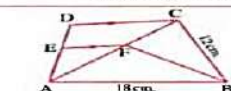
$\therefore \overline{EF} \parallel \overline{DC}$

$$\therefore \frac{AF}{FC} = \frac{AE}{ED} = \frac{3}{2}$$

$$\therefore \frac{AB}{BC} = \frac{18}{12} = \frac{3}{2}$$

$$\therefore \frac{AF}{FC} = \frac{AB}{BC}$$

$\therefore \overline{BF}$ bisects $\angle ABC$ in $\triangle ABC$ (Q.E.D.)



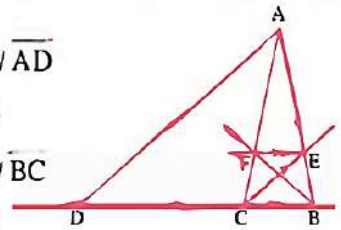
6

In $\triangle ABD$: $\therefore \overline{CE} \parallel \overline{AD}$

$$\therefore \frac{BC}{CD} = \frac{BE}{EA} \quad (1)$$

In $\triangle ABC$: $\therefore \overline{EF} \parallel \overline{BC}$

$$\therefore \frac{CF}{FA} = \frac{BE}{EA} \quad (2)$$



$$\text{From (1), (2) : } \therefore \frac{BC}{CD} = \frac{CF}{FA}$$

$$\therefore AB = CD$$

$$\therefore \frac{BC}{AB} = \frac{CF}{FA}$$

$\therefore \overline{BF}$ bisects $\angle ABC$ in $\triangle ABC$ (Q.E.D.)

7

$\therefore \overline{ED} \parallel \overline{XY} \parallel \overline{BC}$

$$\therefore \frac{EX}{BX} = \frac{DY}{CY} \quad (1)$$

$$\therefore AD \times BX = AC \times EX$$

$$\therefore \frac{EX}{BX} = \frac{AD}{AC} \quad (2)$$

$$\text{From (1), (2) : } \therefore \frac{DY}{CY} = \frac{AD}{AC}$$

$\therefore \overline{AY}$ bisects $\angle CAD$

(Q.E.D.)

8

In $\triangle BFE$: $\therefore \overline{MN} \parallel \overline{BE}$

$$\therefore \frac{BM}{MF} = \frac{EN}{FN}$$

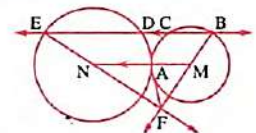
$$\therefore BM = MA, EN = AN$$

$$\therefore \frac{MA}{MF} = \frac{AN}{FN}$$

$$\therefore \frac{MA}{AN} = \frac{MF}{FN}$$

$\therefore \overline{FA}$ bisects $\angle MFN$

(Q.E.D.)



9

$$\therefore \frac{DB}{BE} = \frac{DC}{CE}$$

$\therefore \overline{CB}$ bisects $\angle DCE$ (1)

$\therefore \overline{AB}$ is a diameter of the circle

$$\therefore m(\angle ACB) = 90^\circ$$

$$\therefore \overline{AC} \perp \overline{BC} \quad (2)$$

From (1), (2) : $\therefore \overline{CA}$ bisects $\angle FCE$

(angle bisectors are perpendicular)

(Q.E.D. 1)

$$\therefore \frac{DA}{AE} = \frac{DC}{CE}$$

$$\therefore \frac{DB}{BE} = \frac{DC}{CE}$$

$$\therefore \frac{DA}{DB} = \frac{AE}{BE}$$

$$\therefore \frac{AD}{AE} = \frac{DB}{BE}$$

(Q.E.D. 2)

